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Prompt Leptons in Cosmic Rays.

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Summary. — Energy spectra and zenith-angle distributions of cosmic-ray muons, neutrinos and antineutrinos of prompt generation for energy interval $(1 \div 10^6)$ TeV are calculated. For calculations of differential cross-sections of $D^{\pm}$, D^0, $\overline{D}^0$ and Λ_c production in $\mathcal{N}\mathcal{N}$ and $\pi\mathcal{N}$ interactions the recombination quark-parton model (RQPM) is used. Accounting of nuclear effects is done by using the additive quark model and the optical model of nucleus. Detailed comparison of results obtained in RQPM with corresponding predictions of quark-gluon string model (MQGS) is carried out. Dynamics of semi-leptonic D- and Λ_c-decays and energy losses of muons in the atmosphere are taken into account. Calculations of hadronic cascades in the atmosphere are done with accounting of growth with energy of total inelastic hadron-nucleus cross-sections, steepening of primary cosmic-ray spectrum and processes of pion regeneration. The comparison of our calculations with experimental data and with calculations of other authors is given.

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1. – Introduction.

The object of the present work is the calculation of energy spectra and angular distributions of prompt muons, neutrinos and antineutrinos for energy interval $(1 \div 10^6)$ TeV. The results of numerous works (1-12), devoted to the same problem, are greatly different from one another. The recent data of several

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arrays (KGF, Moscow University X-ray chambers, Baksan Muon Telescope and others) show that the contribution of prompt muons is noticeable on energies above several tens TeV (see [11] and also [4,10]). Future large underground and underwater detectors (see review [13]) will permit to study the region of higher energies (up to $\sim 10^6$ TeV) where prompt muons and neutrino surely dominate.

Almost all previously calculations of prompt lepton fluxes (except [12]) did not take into account the differences in production cross-section of $D^{\pm}$-, D^0-, $\overline{D}^0$-mesons and dynamics of three-particle decay of D and Λ_c. Such an approach contains additional shortcomings connected with the necessity of averaging of production cross-sections and semi-leptonic branching ratios of D-mesons. Moreover, one cannot calculate in this case yields of neutrino and antineutrino separately though it is necessary for exact predictions of the number of events in detectors.

In present work the calculations of inclusive production cross-sections of $D^{\pm}$, D^0, $\overline{D}^0$, Λ_c in pA, nA, πA collisions are carried out for two alternative models, RQPM and MQGS. The recombination model which is used here has been developed in the author's work [14]. Formulae for the cross-sections of charm production in MQGS model are contained in [15].

It is assumed in RQPM that inside hadrons there is an unperturbative sea of c-quarks with rigid momentum spectra. The distributions of c-quarks are

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calculated by using a modified Kuti-Weisskopf model. Inclusive spectra of charmed particles are determined by two- and three-particle distributions of quarks in projectile hadron and recombination probabilities.

MQGS is based on $1/N_f$ expansion in QCD and on theory of up-critical pomeron. Inclusive spectra of particles producted in hadron-nucleus interactions are determined, in contrast with RQPM, by fragmentation functions which, in turn, are calculated by using Regge-pole theory.

Differential widths of semi-leptonic decays are calculated with the help of conserved vector current hypothesis and $SU(4)$-symmetry relations with approximation of constant form factors.

For calculation of the hadronic cascade in the atmosphere we use the results of work ([16]), where nonpower character of primary spectrum, growth with energy of inelastic hadron-nucleus cross-sections and regeneration processes are taken into account. The calculation of separate components (p, n, π^+ and π^-) of the cascade is necessary because the production cross-sections of charmed particles depends on the kind of projectile hadron.

2. – Calculation of hadronic component.

2'1. *Nucleon spectrum on the border of the atmosphere.* – We shall use the approximation of integral primary spectrum supposed in work ([17]) for the energy region $10^3\,\mathrm{GeV} \leqslant \mathcal{E} \leqslant 10^{10}\,\mathrm{GeV}$ ($\mathcal{E}$ is the energy per particle in GeV):

$$(2.1)\qquad \begin{cases} F(\geqslant \mathcal{E}) = F_0\, \mathcal{E}^{-\gamma} \sum\limits_A B_A (1 + \delta_A\, \mathcal{E}/A)^{0.4}, \\ F_0 = 1.16 \cdot 10^4\,\mathrm{m^{-2}\,s^{-1}\,sr^{-1}}, \qquad \gamma = 1.62 \pm 0.03\,. \end{cases}$$

The values of parameters B_A for standard groups of nuclei are presented in table I. Parameters δ_A which characterize the region of primary spectrum steepening are:

$$(2.2)\qquad F\colon \delta_1 = 6 \cdot 10^{-7}, \;\; \delta_{A\geqslant 4} = 10^{-5}; \qquad D\colon \delta_1 = 3 \cdot 10^{-6}, \;\; \delta_{A\geqslant 4} = 6 \cdot 10^{-6}.$$

TABLE I. – *Parameters of the primary cosmic-ray spectrum* (eq. (2.3)).

A	1	4	15	26	51
B_A (%)	40 ± 3	21 ± 3	14 ± 3	13 ± 3	12 ± 4
C_A	1	0.222	0.0653	0.0431	0.0262

([16]) A. N. WALL, V. A. NAUMOV and S. I. SINEGOVSKY: *Yad. Fiz.*, 44, 1240 (1986).
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«F-model» is based on the assumption that the cause of primary spectrum change is energy losses of proton and nuclei in pγ- and Aγ- collisions in cosmic source. «D-model» corresponds to the assumption about the connection of observed spectrum steepening with the dependence of the diffusion coefficient of particles in galactic magnetic fields on energy.

Transforming (2.1) to the equivalent spectrum of nucleons, we obtain the following expression for the differential spectrum (E is the energy in units of GeV/nucleon):

$$\text{(2.3)}\qquad \begin{cases} I(E) = \sum\limits_A I_A(E)\,, \\ I_A(E) = I_0 E^{-(\gamma+1)} C_A (1+\delta_A E)^{-0.4}\left(1 + \dfrac{0.4\gamma^{-1}\delta_A E}{1+\delta_A E}\right), \\ I_0 = 7.51 \cdot 10^3\,\mathrm{m^{-2}\,s^{-1}\,sr^{-1}}\,(\mathrm{GeV/nucleon})^{-1}\,, \qquad C_A = (B_A/B_1)A^{1-\gamma}\,. \end{cases}$$

We will need below the asymptotic formulae for nucleon spectrum, which can be obtained from (2.3) for two energy regions:

$$\text{(2.4)}\qquad I_A(E) = \begin{cases} C_A \cdot I_0 E^{-2.62}\,, & E \ll 6.5/\delta_A\ \mathrm{GeV/nucleon} \\ 1.25\, C_A \cdot \delta^{-0.4} I_0 E^{-3.02}\,, & E \gg 0.6/\delta_A\ \mathrm{GeV/nucleon}\,. \end{cases}$$

For energies $\sim(10 \div 500)$ TeV/nucleon spectrum (2.1) does not contradict the results of experiment JACEE ([18]), but does not agree with the data of Moscow University group ([19]) (the spectrum of work ([19]) for primary protons with $E_\mathrm{p} > 10$ TeV is much more steep than spectrum (2.1)). For superhigh energies both variants of spectrum approximation (see (2.2)) are in agreement (within experimental errors) with the recent results of Akeno group ([20]).

2'2. *Total inelastic cross-sections.* – For accounting the energy dependence of high-energy hadron interaction cross-sections we use the model of elastic

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amplitude ([21]), (see also ([16])) based on the concept of double pomeron with supercritical intercept. In this model one obtains the following expressions for total and inelastic pp cross-sections (for $s \gg s_0 \simeq 100\,\mathrm{GeV}^2$):

$$\sigma_{\mathrm{tot}}(s) = 16\pi\alpha'\,\lambda^{-1}\ln(1+g(s))(1+\lambda\ln(s/s_0))\,, \tag{2.5}$$

$$\sigma_{\mathrm{inel}}(s) = 16\pi\alpha'\,\lambda^{-1}g(s)(1+g(s))^{-1}(1+\lambda\ln(s/s_0))\,. \tag{2.6}$$

These expressions describe quite well the available accelerator data up to energies of Sp$\bar{\mathrm{p}}$S-collider ([22]) if the following values of parameters are used ([21]): $\alpha' = 0.25\,\mathrm{GeV}^{-2}$, $\lambda = 0.07$, $g(s) = g_0(s/s_0)^{\delta}$, $g_0 = 0.46$, $\delta = 0.06$. The corresponding pA cross-sections (obtained by Glauber technique) well agree with the recent results of Fly's Eye ([23]) and Yakutsk ([24]) for inelastic cross-section of p-air interaction $\sigma^{\mathrm{in}}_{\mathrm{pA}}(E)$ up to $E \simeq 3\cdot 10^{10}\,\mathrm{GeV}$, while the fit of Akeno data ([25]), $\sigma^{\mathrm{in}}_{\mathrm{pA}} = 290\,(E/1\,\mathrm{TeV})^{0.06\pm0.005}\,\mathrm{mb}$, gives more strong rise of cross-section in all energy regions. With sufficient accuracy the result of the recalculation can be approximated by equation $\sigma^{\mathrm{in}}_{\mathrm{pA}}(E) = 275\,(1+0.07\ln(E/1\,\mathrm{TeV}))\,\mathrm{mb}$. This approximation numerically is in good agreement with the MQGS calculation of $\sigma^{\mathrm{in}}_{\mathrm{pA}}$ for $E \geqslant 1\,\mathrm{TeV}$ ([26]).

The very important consequence of the model is the asymptotic equality of inelastic cross-sections for any hadrons ([16]). Therefore we assume for $E > E_0$

$$\begin{cases} \sigma^{\mathrm{in}}_{\mathrm{fA}}(E) = \sigma_{\mathrm{fA}} + \sigma_{\mathrm{A}}\ln(E/E_0) \quad \text{for} \quad \mathrm{f} = \mathcal{N}, \pi, \mathrm{D}, \dots\,, \\ \sigma_{\mathrm{A}} = 19\,\mathrm{mb}\,, \qquad E_0 = 10^3\,\mathrm{GeV}\,. \end{cases} \tag{2.7}$$

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For πA interaction $\sigma_{\pi A} = 212\,\text{mb}$ that agrees with the experimental value $\sigma^{\text{in}}_{\text{pA}}/\sigma^{\text{in}}_{\pi\text{A}} = 1.3$ at accelerator energies. The values σ_{DA} and $\sigma_{\Lambda_c \text{A}}$ are presented in table IV (see subsect 2'4).

2'3. *Pion-nucleon cascade model.* – The main channel of charmed hadron production in the atmosphere is the interactions of cosmic-ray protons, neutrons and nuclei with nuclei of air atoms. Of course it is desirable also to value (to estimate) the contribution of πA interactions to charm production in the atmosphere, especially because the inclusive spectra of charmed particles producted in $\pi\mathcal{N}$-collisions are rather flat in RQPM ([14]). The role of pions can be important at very high energies only, so we will include pions to cascade calculation scheme treating them as stable particles. Our estimates show that the contributions of KA interactions and $\text{K}^0_{\text{s}} \to 2\pi$ decays are very small and can be safely neglected.

The pion-nucleon cascade calculation is based on cascade model supposed in ([16]) which describes rather well experimental data on hadron spectra for various atmosphere depths and for energies from 10^3 up to $\sim 6 \cdot 10^5\,\text{GeV}$. We enumerate below the main assumptions of this model.

1) Nucleus component of primary spectrum is replaced by superposition of free nucleons with intensity on the border of the atmosphere described by formulae (2.2)-(2.4). We divide the entire spectrum into two energy intervals: $E < E_{\text{b}}$ and $E > E_{\text{b}}$ (where $E_{\text{b}} = 1.9 \cdot 10^6\,\text{GeV}$ in F-model and $E_{\text{b}} = 5.2 \cdot 10^5\,\text{GeV}$ in D-model). For each interval we assume according to (2.4) the power form of the spectrum. So the differential energy spectra of protons and neutrons on the border of the atmosphere ($h = 0$) are given by relations

$$p(E, h = 0) = I_{\text{p}}(E) + I_{\text{N}}(E), \qquad n(E, h = 0) = I_{\text{N}}(E),$$

$$I_{\text{p}}(E) = I_{A=1}(E), \qquad I_{\text{N}}(E) = 1/2 \sum_{A \geq 4} I_A(E).$$

2) One assumes that there is Feynman scaling in the fragmentation region, so the invariant inelastic cross-sections $E\text{d}^3\sigma^A_{fi}/\text{d}^3p\,(f, i = \mathcal{N}, \pi)$ do not depend on energy. One should note that the violations of scaling predicted by some models (see, *e.g.*, ([26,27])) affect rather weakly the values of partial moments

$$Z_{fi}(\gamma) = \int_0^1 \text{d}x\, x^{\gamma-1} w_{fi}(x), \qquad w_{fi}(x) = \frac{\pi}{\sigma_{iA}} \int \text{d}p^2_{\text{T}} \frac{E}{p_{\text{L}}} \left(E \frac{\text{d}^3 \sigma^A_{fi}}{\text{d}^3 p} \right). \tag{2.8}$$

The values of these moments are mostly essential for calculation of hadron energy spectra on depths $h < (500 \div 600)\,\text{g} \cdot \text{cm}^{-2}$ in the atmosphere ([16]).

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The system of kinetic equations which takes into account the processes of regeneration and production of $\mathcal{N}\mathcal{N}$ pairs in πA interactions is:

$$[\partial/\partial h + 1/\lambda_f(E)]f(E,h) = \sum_i (1/\lambda_i^0) \int_0^1 dx\, x^{-2} w_{fi}(x)\, i(E x^{-1}, h). \tag{2.9}$$

Here $f(E,h)\,dE$ $(i(E,h)\,dE)$ is the intensity of particles $f(i)$ with energies from E to $E+dE$ on depth h;

$$1/\lambda_f(E) = N_0 \sigma_{fA}^{in}(E), \qquad 1/\lambda_i^0 = 1/\lambda_i(E_0) = N_0 \sigma_{iA}$$

$i, f = \mathrm{p}, \mathrm{n}, \pi^+, \pi^-$; N_0 is a number of nuclei per 1g of air.

The exact solution of system (2.7) can be obtained only with the neglect of nucleon production in πA interactions. In the general case one can find the approximate solution in form of expansion on parameter μh $(\mu = N_0 \sigma_A = 0.07/\lambda_N^0)$.

For power primary spectrum one obtains the following expressions for differential energy spectra of $\mathcal{N}$- and π-component of cascade:

$$\begin{cases} p(E,h) = 1/2\,(N^+(E,h) + N^-(E,h)), \\ n(E,h) = 1/2\,(N^+(E,h) - N^-(E,h)), \\ \pi^\pm(E,h) = 1/2\,(\Pi^+(E,h) \pm \Pi^-(E,h)), \end{cases} \tag{2.10}$$

$$\begin{cases} N^\nu(E,h) = (1/2j^\nu)[I_p(E) + \delta^\nu I_N(E)] \sum_\alpha (j^\nu + \alpha)\cdot \\ \qquad\qquad \cdot \exp[-\mu_\alpha^\nu(E)h](1 + O(\mu h)), \\ \Pi^\nu(E,h) = (1/2j^\nu)[I_p(E) + \delta^\nu I_N(E)] \sum_\alpha (-\alpha \delta_N^\nu)\cdot \\ \qquad\qquad \cdot \exp[-\mu_\alpha^\nu(E)h](1 + O(\mu h)). \end{cases} \tag{2.11}$$

Here and below Greek indexes $(\nu, \alpha \ldots)$ mean + and $-$; $\delta^+ = 2$, $\delta^- = 0$,

$$\mu_\alpha^\nu(E) = \frac{1+\alpha j^\nu}{2\Lambda_{\mathcal{N}}^\nu(E)} - \frac{1-\alpha j^\nu}{2\Lambda_\pi^\nu(E)},$$

$$j^\nu = \sqrt{1 + \delta_{\mathcal{N}}^\nu \delta_\pi^\nu} \simeq 1 + 1/2\,\delta_{\mathcal{N}}^\nu \delta_\pi^\nu, \qquad \delta_{\mathcal{N}}^\nu = Z_{\pi\mathcal{N}}^\nu \Lambda_-^\nu / \lambda_{\mathcal{N}}^0, \qquad \delta_\pi^\nu = z_{\mathcal{N}\pi}^\nu \Lambda_-^\nu / \lambda_\pi^0,$$

$$1/\Lambda_f^\nu(E) = 1/\lambda_f(E) - Z_{ff}^\nu/\lambda_f^0, \qquad 1/\Lambda_-^\nu = 1/2\,(1/\Lambda_{\mathcal{N}}^\nu(E_0) - 1/\Lambda_\pi^\nu(E_0)),$$

$$Z_{\mathcal{N}\mathcal{N}}^\nu = Z_{pp} + \nu Z_{pn} = Z_{nn} + \nu Z_{np}, \qquad Z_{\pi\pi}^\nu = Z_{\pi^+\pi^+} + \nu Z_{\pi^+\pi^-} = Z_{\pi^-\pi^-} + \nu Z_{\pi^-\pi^+},$$

$$Z_{\mathcal{N}\pi}^\nu = Z_{p\pi^+} + \nu Z_{p\pi^-} = Z_{n\pi^-} + \nu Z_{n\pi^+}, \qquad Z_{\pi\mathcal{N}}^\nu = Z_{\pi^-p} + \nu Z_{\pi^+n} = Z_{\pi^-n} + \nu Z_{\pi^-p}.$$

Since the values $Z_{\mathcal{N}\pi}^{\nu}$ are small one can use approximations

$$\mu_{+}^{\nu}(E) \simeq 1/\Lambda_{\mathcal{N}}^{\nu}(E) + \delta_{\mathcal{N}}^{\nu}\delta_{\pi}^{\nu}/(2\Lambda_{-}^{\nu}),$$

$$\mu_{-}^{\nu}(E) \simeq 1/\Lambda_{\pi}^{\nu}(E) - \delta_{\mathcal{N}}^{\nu}\delta_{\pi}^{\nu}/(2\Lambda_{-}^{\nu}).$$

It appears, in particular, that in good approximation (neglecting the processes $\pi^{\pm}A \to \mathcal{N}X$) the functions $\mu_{+}^{\pm}(E)$ and $\mu_{-}^{\pm}(E)$ coincide with inverse absorption lengths of nucleons and pions, while $\mu_{+}^{\mp}(E)$ and $\mu_{-}^{\mp}(E)$ are «equalizing lengths» for p-n and π^{+}-π^{-}. It is easy to see that the universality of logarithmic coefficient in (2.7) leads to the same linear rise of spectrum index with depth for all secondary hadrons.

As has been shown in ([16]) the formulae presented above (without correction terms $O(\mu h)$) are valid with good accuracy up to $h \simeq 500\,\mathrm{g\cdot cm^{-2}}$, that is completely enough for our aims.

The values of moments Z_{fi} and parameters Λ_{-}^{ν}, δ_{f}^{ν} and j^{ν} are presented in tables II and III for two values of primary spectrum index. For calculation of

TABLE II. – *Fractional moments $Z_{fi}(\gamma)$ of inclusive distributions of nucleons and pions (reactions* i + A → f + X).

i	f: p	n	π^{+}	π^{-}
$\gamma = 1.62$				
p	0.1990	0.0763	0.0474	0.0318
n	0.0763	0.1990	0.0318	0.0474
π^{+}	0.0070	0.0060	0.1500	0.0552
π^{-}	0.0060	0.0070	0.0552	0.1500
$\gamma = 2.02$				
p	0.1980	0.0585	0.0257	0.0162
n	0.0585	0.1980	0.0162	0.0257
π^{+}	0.0060	0.0040	0.1480	0.0346
π^{-}	0.0040	0.0060	0.0346	0.1480

TABLE III. – *Values of parameters Λ_{-}^{ν}, $\delta_{\mathcal{N},\pi}^{\nu}$ and j^{ν}.*

γ	1.62	2.02	γ	1.62	2.02
$\Lambda_{-}^{+}/\lambda_{\mathcal{N}}^{0}$	17.65	17.43	$\Lambda_{-}^{-}/\lambda_{\mathcal{N}}^{0}$	11.05	11.20
$\delta_{\mathcal{N}}^{+}$	1.398	0.730	$\delta_{\mathcal{N}}^{-}$	0.172	0.106
δ_{π}^{+}	0.177	0.134	δ_{π}^{-}	0.009	0.008
j^{+}	1.117	1.084	j^{-}	1	1

$Z_{fi}(\gamma)$ (except $Z_{p\pi^\pm}$, $Z_{n\pi^\pm}$) we used the parametrization of ISR data from [28,29]. The values $Z_{p\pi^\pm}$ and $Z_{n\pi^\pm}$ were calculated by us from two central moments of inclusive distributions obtained in [27].

2·4. *Energy spectra of* D *and* Λ_c^+. – We neglect the regeneration of short-lived particles (D, Λ_c^+). This is surely justified for $E < E_f^c(\theta)$, where $E_f^c(\theta) = (m_f H_0/\tau_f)\sec\theta^*$ is the critical energy of particle flying at zenith angle θ (m_f is the mass, τ_f is the lifetime, $H_0 = 6.44$ km is the parameter of isothermal atmosphere with $T = 220$ K, $\mathrm{f} = \mathrm{D}^\pm, \mathrm{D}^0, \overline{\mathrm{D}}^0, \Lambda_c$); θ^* is the effective angle coinciding with θ in the limit of flat atmosphere (*).

In this approximation the differential energy spectrum of f-particles is given by the following expression:

$$f(E,h,\theta) = \int_0^h \mathrm{d}t \exp\left[-\frac{h-t}{\lambda_f(E)} - \frac{m_f}{\tau_f E}\int_t^h \frac{\mathrm{d}u}{\rho(u,\theta)}\right] G_f(E,t), \tag{2.12}$$

where $\rho(h,\theta)$ is the air density on the level corresponding to the direction θ and depth h along this direction, and

$$G_f(E,h) = \sum_i \frac{1}{\lambda_i^0}\int_0^1 \mathrm{d}x\, x^{-2} w_{fi}(x)\, i(Ex^{-1},h) \tag{2.13}$$

is the production function of f by nucleons and pions ($i = \mathrm{p}, \mathrm{n}, \pi^+, \pi^-$).

For energy region $E \geqslant E_f^c(\theta)$ the regeneration can, in principle, be important, so the expression (2.12) gives there only the low limit of charmed-hadron spectrum.

Using in (2.12) the standard approximation

$$\rho(h,\theta) = \rho(h\cos\theta^*, 0) = h\cos\theta^*/H_0$$

(θ^* is the zenith angle of penetration of particle on the height H^*), one obtains

$$f(E,h,\theta) = \int_0^h \mathrm{d}t\,(t/h)^{E_f^c(\theta)/E} \exp[-(h-t)/\lambda_f(E)]\, G_f(E,t). \tag{2.14}$$

This expression is simplified in two limit cases: $E \ll \overline{x}_f E_B$ and $E \geqslant E_B$ ($\overline{x}_f$ is the mean part of energy transferred to the particle f in reactions $\mathcal{N}\mathrm{A} \to \mathrm{fX}$ and

([28]) Y. Minorikawa and K. Mitsui: *Proceedings of the XVII International Cosmic Rays Conference*, Vol. **10** (Paris, 1981), p. 353.
([29]) Y. Minorikawa and K. Mitsui: *Lett. Nuovo Cimento*, 41, 353 (1984).
(*) For spherical atmosphere $\sec\theta^* = \sqrt{\pi R_E/2H_0}\,\mathrm{erfc}[x]\exp[x^2]$, $x^2 = (R_E\cos^2\theta + 2H^*)/2H_0$, where R_E is mean radius of Earth, H^* is the height of effective prodiction.

$\pi A \rightarrow fX$). Using (2.10) and (2.11) for intensities of $\mathcal{N}$- and π-components one can present the production function (2.13) in the following form:

$$(2.15)\qquad G_f(E,h) = (2\lambda_{\mathcal{N}}^0)^{-1} \sum_\nu N^\nu(E,h)\, Z_{f\mathcal{N}}^\nu(\gamma+\mu h) + \\ + (2\lambda_\pi^0)^{-1} \sum_\nu \Pi^\nu(E,h)\, Z_{f\pi}^\nu(\gamma+\mu h),$$

where $Z_{f\mathcal{N}}^\nu = Z_{fp} + \nu Z_{fn}$; $Z_{f\pi}^\nu = Z_{f\pi^+} + \nu Z_{f\pi^-}$.

Substituting (2.15) into (2.14) and neglecting in the integration over t the dependence of $Z_{fi}^\nu(\gamma+\mu t)$ on t, we obtain

$$(2.16)\qquad f(E,h,\theta) = (h/2\varepsilon_f(\theta)\,\lambda_{\mathcal{N}}^0) \sum N_f^\nu(E,h,\theta)\, Z_{f\mathcal{N}}^\nu(\gamma+\mu h) + \\ + (h/2\varepsilon_f(\theta)\,\lambda_\pi^0) \sum \Pi_f^\nu(E,h,\theta)\, Z_{f\pi}^\nu(\gamma+\mu h),$$

where $\varepsilon_f(\theta) = E_f^c(\theta)/E + 1$. The functions N_f^ν and Π_f^ν are obtained from the functions N^ν and Π^ν by the following substitution in (2.11):

$$(2.17)\qquad \exp[-\mu_\alpha^\nu(E)h] \rightarrow \\ \rightarrow \Gamma(\varepsilon_f(\theta)+1)\exp[-h/\lambda_f(E)]\,\gamma^*/\varepsilon_f(\theta), [\mu_\alpha^\nu(E) - \lambda_f^{-1}(E)]\,h.$$

Symbol γ^* here designates the incomplete gamma-function [30]. For $E \ll E_f^c(\theta)$ one obtains, as is seen from (2.17),

$$N_f^\nu(E,h,\theta) = N^\nu(E,h), \qquad \Pi_f^\nu(E,h,\theta) = \Pi^\nu(E,h).$$

So the dependence on $\lambda_f(E) = [N_0 \sigma_{fA}^{in}]^{-1}$ vanishes.

The values E_f^c, σ_{fA}, etc. used in calculations are presented in table IV. The height of effective production is calculated by approximation formula [31]

$$H^* = 34 - 10.5\cos\theta^* - 120\cos^2\theta^* - 250\cos^3\theta^*\ \text{km}, \qquad \theta > 70°.$$

TABLE IV. – *Characteristic of charmed hadrons.*

f	m_f(GeV)	$\tau_f(10^{-13}\,\text{s})$	B_f(%)	$E_f^c(0)\,(10^7\,\text{GeV})$	σ_{fA} (mb)
D^+, D^-	1.8693	9.2	18.0	4.3	100
$D^0, \overline{D}^0$	1.8646	4.3	7.0	9.2	100
Λ_c^+	2.2812	2.3	4.5	21.0	200

[30] M. ABRAMOVITS and I. STIGANAM (editors): *Spravochnick po spetsialnym funktsiyam pod red.* (Nauka, Moscow, 1979).
[31] E. V. BUGAEV, V. A. NAUMOV: preprint IYaI AN SSSR II-0537 (Moscow, 1987).

The calculations show that the angular distributions of prompt leptons depend rather weakly on the value H^* even for $\theta \sim 90°$.

3. – Inclusive cross-sections of charm production.

According to experimental data the cross-section of charm production in $\mathcal{N}\mathcal{N}$-collisions is relatively large (~ 1 mb for ISR energies, $\sqrt{s} = 62$ GeV), and inclusive distributions of produced particles on longitudinal momentum are rather flat, expecially in case of charmed barions. It is impossible to explain such behaviour with the help of models based on QCD perturbation theory (32). So it is natural to suppose that at very high energies the processes of charm production in hadronic collisions are not hard (the mass of c-quark is not large enough) and to use for calculations of inclusive charm production cross-sections some nonperturbative models.

Most of nonperturbative quark models of multiple production (of hadrons with small p_T) can be divided into two groups: recombination models and fragmentation models. In the latter models the final stage (hadronization) is described in terms of fragmentation functions. The mechanism of hadronization is connected with break of quark chains (strings). These chains arise between valence and sea quarks of colliding hadrons at the initial stage of interaction. The known models of such type are: dual parton model (33) and model of quark-gluon strings (15).

In recombination models (34-36) the process of hadronization occurs by means of recombination of quarks to hadrons. Here two assumptions are principal: 1) during collision only the slow (wee) partons interact, therefore the inclusive spectra of produced particles (with small p_T and not very small x) are entirely determined by quark distributions inside colliding hadrons (*i.e.* it is assumed that the distributions of fast quarks almost do not change through the collision); 2) the hadronization process can be described by the recombination functions which depend only on quark momenta. The inclusive cross-section of hadron production is written as follows:

$$\mathrm{d}\sigma^{\mathcal{N}}_{fi}/\mathrm{d}x = \sigma_{\mathrm{tot}}(s)\int F\{x_k\}\, R\{x_k\}\, \delta\Big(\sum_k x_k - x\Big) \prod_k \mathrm{d}x_k\,, \tag{3.1}$$

(32) E. V. BUGAEV and E. S. ZASLAVSKAYA: preprint IYaI AN SSSR П-0467 (Moscow, 1986).

(33) A. CAPPELLA, U. SUKHATME, CHUNG-I TAN and J. TRAN THANK VAN: *Phys. Lett. B*, **81**, 68 (1979).

(34) K. P. DAS and R. C. HWA: *Phys. Lett. B*, **68**, 459 (1977).

(35) D. W. DUKE and F. E. TAYLOR: *Phys. Rev. D*, **17**, 1788 (1977).

(36) E. TAKASUGI and X. TATA: *Phys. Rev. D*, **26**, 120 (1982).

where x_k is part of the projectile momentum, which belongs to k-parton, $F(x_k)$ is the two- or three-quark distribution in projectile hadron, $R(x_k)$ is the recombination function for two or three quarks. One must note that usually the momentum-conserving delta-function in integral (3.1) is included in the definition of recombination function $R(x_k)$. This formula is easily obtained if we take into account that (according to assumption 1) the distribution of wee partons is universal and does not correlate with the distribution of fast partons. It is natural to assume that the recombination functions and parton distributions do not depend on energy of projectile particle far from threshold of charm production. If this is so then the dependence of $\mathrm{d}\sigma^{\mathcal{N}}_{fi}/\mathrm{d}x$ on s is entirely determined by the energy dependence of total cross-section $\sigma_{\text{tot}}(s)$ of hadron-nucleon interaction.

In RQPM for the calculation of two- and three-quark distributions the statistical approach is used (see [14] for details). The differential probability of state with n_s sea partons and n_v valence quarks is equal to

$$\text{(3.2)}\qquad \mathrm{d}W_{n_\text{s}} = Z(P)\prod_{k=1}^{n_\text{v}} f^\text{v}_k(x_k)\prod_l \frac{1}{n_l!}\prod_{m=1}^{n_l} f^\text{s}_m(x_m)\prod_{n=1}^{n_\text{s}+n_\text{v}}\mathrm{d}x_n\cdot$$
$$\cdot(x_n^2+x_\text{T}^2)^{-1/2}\,\delta\left(1-\sum_{n=1}^{n_\text{s}+n_\text{v}} x_n\right);\qquad \int\sum_n \mathrm{d}W_n = 1\,,$$

where P is the momentum of projectile hadron, $x_\text{T} = m_\text{T}/P$, m_T is the transverse mass of parton; n_k is number of partons of k-type, $Z(P)$ is the normalization factor, f^v_k and f^s_m are «uncorrelated» parton distributions of valence and sea quarks. Parameters of these distributions are found from comparison of calculations (using formula (3.1)) with experiment. Usual one-particle structure functions of quarks are obtained from (3.2) by integration over all x_k except one and are described through correlation functions $G^p_0(1-x)$ calculated in [37]. The behaviour of structure functions of sea quarks at $x\to 1$ depends (as the analysis of inclusive spectrum data [36] has shown) on quark mass:

$$\text{(3.3)}\qquad \begin{cases} u^\text{s}(x) = f^\text{s}_\text{u}(x)\,G^p_0(1-x)\sim(1-x)^{6.3}\,,\\[2ex] s^\text{s}(x) = f^\text{s}_\text{s}(x)\,G^p_0(1-x)\sim(1-x)^{4.3}\,.\end{cases}$$

It is seen from (3.3) that the distribution of more heavy quarks decreases less steeply with x at $x\to 1$. Analogous results (numerically close to (3.3)) have been obtained in the model of valons [38].

([37]) E. Takasugi, X. Tata, C. Chiu and R. Kaul: *Phys. Rev. D*, **20**, 211 (1979).
([38]) L. Gatignon, Z. Dziembovski, P. A. van der Poel, D. J. Schotanus, A. Stergiou and R. T. Van de Walle: *Z. Phys. C*, **16**, 229 (1983); F. Amiri and P. K. Williams: *Phys. Rev. D*, **24**, 2409 (1981).

We assume that the sea of c-quarks in hadron is essentially nonperturbative and characterizes by flat momentum spectrum. This assumption have been discussed in several works (see, *e.g.*, ([39,40])). It is based on parton concept, according to which in the frame of infinite large momentum the lifetime of fluctuations containing heavy quarks becomes very large. Flatness of heavy-quark spectra follows from a simple picture of nucleon as an aggregate of partons which have approximately equal velocities ([39]) and from calculations of structure functions of strongly bound states ([40]). In this work we use the results of ([14]) for structure functions of c-quarks in nucleon and pion which are presented in fig. 1. The corresponding (to these structure functions) uncorrelated distributions $f_c^s(x)$ in the region $0.1 \leqslant x \leqslant 0.9$ (which is most interesting for us) can be approximated by formulae ([14])

$$f_c^s(x) = 1.10\, c\, x^{-1}(1-x)^{-1.83} \qquad \text{(proton)}, \tag{3.4a}$$

$$f_c^s(x) = 1.54\, c\, x^{-1}(1-x)^{-0.85} \qquad \text{(pion)}. \tag{3.4b}$$

At small x these distributions are proportional to x^{-1} as well as in the case of analogous functions for light sea quarks. But this behaviour contradicts the experimental data at ISR energies (these energies are far from the asymptotic region). Therefore we modify the function (3.4a) by introducing factor $\sqrt{x}$,

$$f_c^{s(\mathrm{mod})}(x) = 1.1\, c\, x^{-0.5}(1-x)^{-1.83}. \tag{3.4c}$$

This modification leads to cross-sections $\mathrm{d}\sigma^{\mathcal{N}}_{\bar{f}i}/\mathrm{d}x$, which agree with experiment at ISR energies $\sqrt{s} = 62\,\mathrm{GeV}$. With increase of energy the function $f_c^s(x)$ must change, passing gradually to the asympotic form of (3.4a). Since the energy at

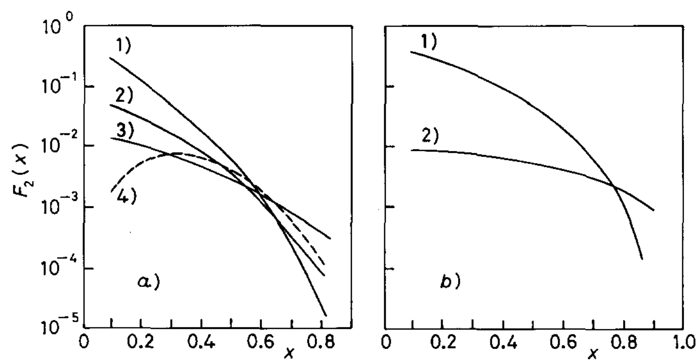

Fig. 1. – Structure functions of sea quarks in nucleon: a) 1) $xu^s(x)$, 2) $xs^s(x)$, 3) $xc^s(x)$, 4) $xc^s(x)$ in the model of ref. ([39]) and in pion: a) 1) $xu^s(x)$, 2) $xc^s(x)$.

([39]) S. J. Brodsky, C. Peterson and N. Sakai: *Phys. Rev. D*, 2745 (1981).
([40]) A. De Rujula and F. Martin: *Phys. Rev. D*, **22**, 1787 (1980).

which asymptotic (scaling) regime begins is unknown it is natural to use in calculation both functions (3.4*a*) and (3.4*c*). Evidently the second choice (eq. (3.4*c*)) leads to predictions only for asymptotically high energies.

The flat structure functions of c-quarks shown in fig. 1 are obtained also in valon model. In this case one must assume that the distribution of c-quarks in valon almost does not depend on part of valon's momentum (*i.e.*, it is approximately uniform). For comparison on the same figure the structure function obtained in «intrinsic charm» model ([39]) is shown.

The constant «c» have been found in ([14]) from experimental data on charm production in deep inelastic scattering. The best agreement with experiment is obtained with $c = 5 \cdot 10^{-3}$.

For recombination functions of quarks to hadrons the following expression are used:

$$(3.5)\qquad R^{\mathrm{D}}(x_1, x_2, x) = \frac{x}{B(a,b)}\left(\frac{x_1}{x}\right)^a \left(\frac{x_2}{x}\right)^b \delta(x_1 + x_2 - 1)\,, \qquad R^{\Lambda_c^+}(x_1, \dots; x) = \frac{x}{B(a, a+b)\,B(a,b)}\left(\frac{x_1 x_2}{x}\right)^a \left(\frac{x_3}{x}\right)^b \delta(x_1 + x_2 + x_3 - 1)\,.$$

Here $B(a, b)$ is the beta-function. In calculations the values $a = 1$, $a/b = m_{\mathrm{u}}/m_{\mathrm{c}} \simeq 1/6$ are used.

The two- and three-particle distributions entering in (3.1) are calculated with help of uncorrelated functions $f_k^l(x)$ ($l = \mathrm{v}, \mathrm{s}$; $k = \mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}$) and correlation functions. For example, two-particle distribution of u- and c-quarks in proton has the following view:

$$(3.6)\qquad F_{\mathrm{u}\bar{\mathrm{c}}}(x_1, x_2) = 2 f_{\mathrm{u}}^{\mathrm{v}}(x_1) f_{\bar{\mathrm{c}}}^{\mathrm{s}}(x_2)\, G_{\mathrm{u}}^{p}(1 - x_1 - x_2) + f_{\mathrm{u}}^{\mathrm{s}}(x_1) f_{\bar{\mathrm{c}}}^{\mathrm{s}}(x_2)\, G_0^{p}(1 - x_1 - x_2)\,.$$

In fig. 2 inclusive cross-sections of D^+, D^0, $\overline{\mathrm{D}}{}^0$ production in pp collisions and inclusive spectra of D^0 and Λ_c^+ predicted by RQPM are given for ISR energies $s = 62$ GeV. Experimental data are taken from works ([41,42]). Comparison of MQGSs predictions with experiment are given in ([15]).

For taking into account of nuclear effect in RQPM we use the additive quark model (AQM). According to AQM the cross-section of process $\mathrm{i} + \mathrm{A} \to \mathrm{f} + \mathrm{X}$ can be expressed through the cross-section of reaction $\mathrm{i} + \mathcal{N} \to \mathrm{f} + \mathrm{X}$ and capture probabilities of valence quarks by target nucleus (which are calculated with the help of the optical model). The results of calculations can be presented in the

([41]) V. BARGER, F. HALZEN and W. Y. KEUNG: *Phys. Rev. D*, 25, 112 (1982).
([42]) M. BASILE, G. CARA ROMEO, L, CIFARELLI, A. CONTIN, G. D'ALI, P. CESARE, B. ESPOSITO, P. GIUSTI, T. MASSAM, R. NANIA, F. PALMONARI, G. SARTORELLI, G. VALENTI and A. ZICHICHI: *Lett. Nuovo Cimento*, 33, 33 (1982).

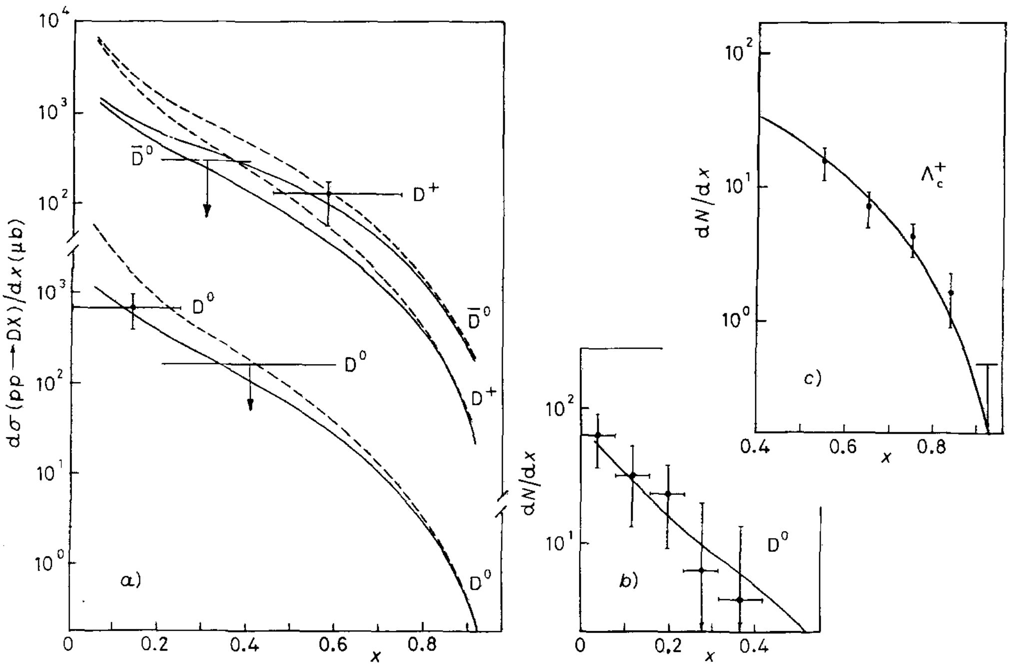

Fig. 2. – Inclusive cross-sections and spectra (in arbitrary units) of charm production at $\sqrt{2} = 62\,\mathrm{GeV}$. *a*) Cross-sections of D^+, D^0 and $\bar{D}^0$ production. *b*) Inclusive spectrum of D^0 for a reaction $pp \to D^0X$. *c*) Inclusive spectrum of Λ_c^+ for a reaction $pp \to \Lambda_c^+X$. Solid curve: the calculation in RQPM with modified distribution of sea c-quarks (eq. (3.4*c*)); dashed curves: the calculation with asymptotical distribution (eq. (3.4*a*)). Experimental data are taken from works [41,42].

form

$$d\sigma_{fi}^A/dx = A^{\alpha(x)}\, d\sigma_{fi}^N/dx\,.$$

The dependences $\alpha = \alpha(x)$ are shown in fig. 3. It was assumed in this calculation that the captured («wounded») quarks do not take part in recombination. It is clear that in reality some portion of wounded quarks will participate in recombination, and taking into account this effect will increase the cross-section $d\sigma_{fi}^A/dx$. For estimation of this effect we obtain the upper limit of $\alpha(x)$, assuming that all valence quarks of projectile can recombinate. This estimation gives [14]

$$\alpha \leq 0.85 \quad \text{for} \quad \pi A \to DX\,, \qquad \alpha \leq 0.785 \quad \text{for} \quad pA \to D(\Lambda_c^+)X\,.$$

One can conclude that the uncertainty in A-dependence does not exceed $(10 \div 15)\%$. Therefore for calculations with MQGS we assumed that α is constant

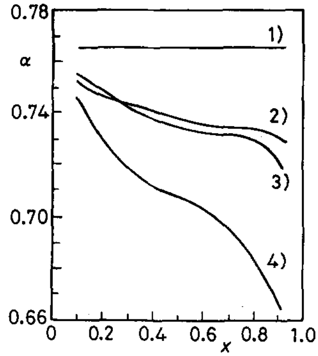

Fig. 3. – $\alpha(x)$-dependences for reactions $\pi A \to DX$ (1)), $pA \to \bar{D}^0 X$ (2)), $pA \to D^- X$ (3)), $pA \to \Lambda_c^+ X$ (4)).

and equal to 0.72 for all reactions. It does not contradict recent data of Fermilab, CERN and Serpukhov (see ([43])).

Partial moments $Z_{fi}(\gamma)$, defined by (2.8), are calculated in RQPM with using of formula (2.5) for total cross-section. The results can be approximated with good

TABLE V. – *Parameters* $F_{fi}(\gamma, E_\gamma)$ *and* ξ_γ *(in brackets) of approximation formulae* (3.7) *for fractional moments* $Z_{fi}(\gamma, E)$ *calculated in framework of* RQPM *and* MQGS.

i	f: D^+	D^-	D^0	$\bar{D}^0$	Λ_c^+
RQPM, $\gamma = 1.62\,(E_\gamma = 1\,\text{TeV},\ \xi_\gamma = 0.096)$					
p	$4.6 \cdot 10^{-4}$	$6.5 \cdot 10^{-4}$	$3.8 \cdot 10^{-4}$	$6.9 \cdot 10^{-4}$	$4.9 \cdot 10^{-4}$
π^+	$1.3 \cdot 10^{-3}$	$9.0 \cdot 10^{-4}$	$9.0 \cdot 10^{-4}$	$1.3 \cdot 10^{-3}$	$6.0 \cdot 10^{-4}$
RQPM, $\gamma = 2.02\,(E_\gamma = 10^3\,\text{TeV},\ \xi_\gamma = 0.076)$					
p	$5.4 \cdot 10^{-4}$	$7.9 \cdot 10^{-4}$	$4.5 \cdot 10^{-4}$	$8.6 \cdot 10^{-4}$	$6.2 \cdot 10^{-4}$
π^+	$1.8 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$	$7.9 \cdot 10^{-4}$
MQGS, $\gamma = 2.02\,(E_\gamma = 10^3\,\text{TeV})$					
p	$6.5 \cdot 10^{-5}$	$9.9 \cdot 10^{-5}$	$7.1 \cdot 10^{-5}$	$2.1 \cdot 10^{-4}$	$9.5 \cdot 10^{-4}$
	(0.050)	(0.046)	(0.050)	(0.044)	(0.041)
n	$7.1 \cdot 10^{-5}$	$1.9 \cdot 10^{-4}$	$6.5 \cdot 10^{-5}$	$1.2 \cdot 10^{-4}$	$9.5 \cdot 10^{-4}$
	(0.050)	(0.045)	(0.050)	(0.045)	(0.041)
π^+	$5.5 \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$	$1.5 \cdot 10^{-5}$
	(0.041)	(0.048)	(0.048)	(0.041)	(0.035)
π^-	$1.4 \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$	$1.5 \cdot 10^{-5}$
	(0.048)	(0.041)	(0.041)	(0.048)	(0.035)

([43]) M. MACDERMOTT and S. REUCROFT: *Phys. Lett. B*, 184, 108 (1987).

accuracy ($(2 \div 3)\%$) by the following expression:

$$Z_{fi}(\gamma, E) = Z_{fi}(\gamma, E_\gamma)(E/E_\gamma)^{\xi_\gamma}, \tag{3.7}$$

where E is the energy of secondary particle $\mathrm{f}\,(\mathrm{f} = \mathrm{D}^\pm, \mathrm{D}^0, \overline{\mathrm{D}}{}^0, \Lambda_c^+)$,

$$E_\gamma = \begin{cases} 10^3\,\mathrm{GeV}, & \text{for} \quad \gamma = 1.62 \\ 10^6\,\mathrm{GeV}, & \text{for} \quad \gamma = 2.02 \end{cases} \qquad \xi_\gamma = 0.177 - 0.05\,\gamma.$$

Parameters $Z_{fi}(\gamma, E_\gamma)$ for $i = \mathrm{p}, \pi$ are presented in table V.

For $i = \mathrm{n}, \pi^-$ we use the relations following from isotopic considerations (analogous to the case of $\mathcal{N}\mathcal{N}$ and $\pi\mathcal{N}$ interactions):

$$Z_{\mathrm{D^+n}} = Z_{\mathrm{D^0p}}; \quad Z_{\mathrm{D^-n}} = Z_{\bar{\mathrm{D}}^0\mathrm{p}}; \quad Z_{\mathrm{D^0n}} = Z_{\mathrm{D^+p}}; \quad Z_{\bar{\mathrm{D}}^0\mathrm{n}} = Z_{\mathrm{D^-p}}; \quad Z_{\Lambda_c^+\mathrm{n}} = Z_{\Lambda_c^+\mathrm{p}}; \tag{3.8}$$

$$Z_{\mathrm{D}^+\pi^+} = Z_{\bar{\mathrm{D}}^0\pi^-} = Z_{\mathrm{D}^0\pi^+}, \qquad Z_{\mathrm{D}^-\pi^+} = Z_{\mathrm{D}^0\pi^-} = Z_{\bar{\mathrm{D}}^0\pi^+}, \qquad Z_{\Lambda_c^+\pi^-} = Z_{\Lambda_c^+\pi^+}. \tag{3.9}$$

For comparison the partial moments have been calculated also with assumption of Feynman scaling (in this case $Z_{fi} = Z_{fi}^{\mathrm{sc}}(\gamma)$, see table VI).

In MQGS formula (3.7) is valid only in very high energy region ($E \gtrsim \overline{x}_f E_\mathrm{B}$). In contrast with RQPM, parameters ξ_γ are, generally speaking, different for different reactions $\mathrm{iA} \to \mathrm{fX}$ (see table V).

TABLE VI. – *Fractional moments* $Z_{fi}^{\mathrm{sc}}(\gamma)$ *(miltiplied by factor* 10^4*) of inclusive distributions of charmed particles, calculated in* RQPM *on the supposition of Feynman scaling.*

i	f: D^+	D^-	D^0	$\overline{\mathrm{D}}{}^0$	Λ_c^+
$\gamma = 1.62$					
p	4.02	5.74	3.35	6.11	4.36
n	3.35	6.11	4.02	5.74	4.36
π^+	12.0	8.09	8.09	12.0	5.44
π^-	8.09	12.0	12.0	8.09	5.44
$\gamma = 2.02$					
p	2.53	3.75	2.11	4.07	2.95
n	2.11	4.07	2.53	3.75	2.95
π^+	8.51	5.56	5.56	8.51	3.79
π^-	5.56	8.51	8.51	5.56	3.79

4. – Muons and neutrino of prompt production.

4·1. *Formulae for prompt lepton spectra.* – The kinetic equation for prompt muons in the atmosphere in the approximation of continuous energy losses is:

$$\frac{\partial}{\partial h}\mu_{\mathrm{pr}}(E,h,\theta)=\frac{\partial}{\partial E}[\beta(E)\mu_{\mathrm{pr}}(E,h,\theta)]+\sum_{\mathrm{f}} G_\mu^{\mathrm{f}}(E,h,\theta)\,. \tag{4.1}$$

Here $\beta(E)=-\mathrm{d}E/\mathrm{d}h=\alpha(E)+b(E)E$ is muon energy losses (ionization, radiation and photonuclear ones); G_μ^{f} are production functions of leptons in semi-leptonic decays of $\mathrm{f}\,(\mathrm{f}=\mathrm{D}^\pm,\mathrm{D}^0,\overline{\mathrm{D}}{}^0,\Lambda_c^+)$,

$$G_\mu^{\mathrm{f}}(E,h,\theta)=\frac{B_{\mathrm{f}}m_{\mathrm{f}}}{\tau_{\mathrm{f}}\rho(h,\theta)}\int\limits_{E_{\mathrm{f}}^-}^{E_{\mathrm{f}}^+}\frac{\mathrm{d}E_{\mathrm{f}}}{E_{\mathrm{f}}}W_\mu^{\mathrm{f}}(E,E_{\mathrm{f}})f(E_{\mathrm{f}},h,\theta)\,,$$

$B_{\mathrm{f}}=B(\mathrm{f}\to\mu\nu\mathrm{X})$ is the branching ratio of quasi-three-particle decay (X is any group of hadrons),

$$E_{\mathrm{f}}^\pm=\frac{m^2}{m_\mu^2}E\,[1\pm\sqrt{(1\pm m_\mu^2/E^2)(1+m_\mu^2m_{\mathrm{f}}^2/m^4)}]\,,$$

$$m^2=1/2\,(m_{\mathrm{f}}^2-m_{\mathrm{X}}^2+m_\mu^2)$$

($m_{\mathrm{f}},m_{\mathrm{X}}$ and m_μ are masses of f, X and μ, correspondingly), W_μ^{f} is the energy spectrum of muons in decay $\mathrm{f}\to\mu\nu\mathrm{X}$ in laboratory frame.

The solution of eq. (4.1) is given by

$$\mu_{\mathrm{pr}}(E,h,\theta)=\sum_{\mathrm{f}}\int\limits_0^h \mathrm{d}t\,\beta^{-1}(E)\beta(E_t)G_\mu^{\mathrm{f}}(E_t,h-t,\theta)\,, \tag{4.2}$$

where E_t is a root of equation $R_\mu(E_t,E)=t$, and

$$R_\mu(w,E)=\int\limits_E^w \mathrm{d}E'\,\beta^{-1}(E')$$

is a range of muon with initial energy w and final energy E.

We limit ourselves to the approximation $\alpha=\mathrm{const}$, $b=\mathrm{const}$. As our analysis shows, the accounting of energy dependence of α and b leads to maximum corrections $\sim 5\%$ (for horizontal direction). In this approximation

$$E_t=(E+\alpha/b)\exp[bt]-\alpha/b\,,\qquad \beta(E_t)=\exp[bt]\beta(E) \tag{4.3}$$

and we use the concrete values

$$\alpha=2.3\,\mathrm{MeV\,cm^2/g}\,,\qquad b=3.5\cdot10^{-6}\,\mathrm{cm^2/g}\,.$$

The energy spectra of prompt anti(neutrino) are:

$$\overset{(-)}{\nu}_{\rm pr}(E,h,\theta)=\sum_{\rm f}\int_0^h {\rm d}t\, G_\nu^{\rm f}(E,t,\theta)\,,$$

$\rm f = D^+, D^0, \overline{D}^0, \Lambda_c^+$ for ν and $f = \rm D^-, D^0, \overline{D}^0$ for $\bar{\nu}$;

$$G_\nu^{\rm f}(E,h,\theta)=\frac{B_{\rm f} m_{\rm f}}{\tau_{\rm f}\rho(h,\theta)}\int_{E_{\rm f}^{\rm min}}^{\infty}\frac{{\rm d}E_{\rm f}}{E_{\rm f}}W_\nu^{\rm f}(E,E_{\rm f})f(E_{\rm f},h,\theta)\,, \tag{4.4}$$

where

$$E_{\rm f}^{\rm min}=1/2\,(m_{\rm f}^2 E/m^2+m^2/E)\,.$$

4.2. *Differential widths of charmed hadron decays.* – For calculating decay spectra we use the phenomenological approach. The matrix element of decay $\rm D\to \ell\nu_\ell X$ can be written as

$$|\mathcal{M}|=(G_{\rm F}\cos\theta_{\rm C}/\sqrt{2})|f^+(q^2)|(p_{\rm D}+p)_\mu j^\mu\,, \tag{4.5}$$

where $G_{\rm F}$ is Fermi constant, $\theta_{\rm C}$ is Cabibbo angle, $q=p_{\rm D}-p=k_1+k_2$, $p_{\rm D}$ and p are four-momenta of D and X, correspondingly, k_1 and k_2 are lepton four-momenta, j^μ is leptonic $(V-A)$-current. It is assumed in (4.5) that X is a pseudoscalar meson and that second form factor f^- is equal to zero (the latter enters the amplitude only in the terms proportional to $m_{\rm e}$). We use the approximation $f^+=$ const, changing $f^+(q^2)$ to $f^+(\overline{q^2})$ and using $\overline{q^2}=2(m_{\rm D}^2/3-m_{\rm X}^2)$ (it corresponds to the assumption that meson X takes away 1/3 of D energy in laboratory frame).

From (4.5) one obtains the following expression for lepton energy spectrum (for ℓ or ν_ℓ) in laboratory frame:

$$W_{\ell(\nu_\ell)}^{\rm D}(E,E_0)=[1/\Gamma({\rm D}\to\ell\nu_\ell{\rm X})]\,{\rm d}\Gamma^{\rm D}/{\rm d}E\,, \tag{4.6}$$

$$\begin{aligned}{\rm d}\Gamma^{\rm D}/{\rm d}E=(4\pi)^{-3}G_{\rm F}^2\cos^2\theta_{\rm C}|\overline{f^+}|^2 m_{\rm D}E_0^{-2}\{(1-r^2)(1-5r^2-2r^4)/6\,+\\ +\,r^4x-(1/2-r^2)x^2+x^3/3+r^4\ln[(1-x)/r^2]\}\,,\end{aligned} \tag{4.7}$$

where E and E_0 are energies of lepton and D-meson, correspondingly, $x=E/E_0$, $r=m_{\rm X}/m_{\rm D}$. Integrating (4.7) over E by using of the following limits $(\mu^2\equiv(m_{\rm D}^2-m_{\rm X}^2)/2)$:

$$E_{\rm min}=\mu^2(E_0-|\boldsymbol{p}_0|)/m_{\rm D}^2\simeq\mu^2/E_0\,,$$

$$E_{\rm max}=\mu^2(E_0+|\boldsymbol{p}_0|)/m_{\rm D}^2\simeq(1-r^2)E_0\,,$$

we obtain the partial width of decay in the centre-of-mass frame

$$\Gamma(\mathrm{D}\to \ell\nu_\ell \mathrm{X}) = (4\pi)^{-3} G_\mathrm{F}^2 \cos^2\theta_\mathrm{C}\, m_\mathrm{D}^5 |\overline{f^+}|^2 [(1-8r^2+8r^6-r^8)/12 - r^4 \ln r^2]\,. \tag{4.8}$$

Passing on to inclusive decay $\mathrm{D}\to \ell\nu_\ell +$ all we keep m_X as a free parameter which we choose from accordance of calculation (using (4.8)) and experimental data on partial width ($B_\mathrm{D}/\tau_\mathrm{D}$) of inclusive semi-leptonic decay (see table IV). Since $\overline{f}^+$ depends on m_X weakly, the exact form of q^2-dependence of f^+ is not very essential. We assume (following to work ([44])) that

$$f^+ = (1 - q^2/m_\mathrm{F}^{2*})^{-1}\,, \qquad m_\mathrm{F}^* = 2.2\,\mathrm{GeV}\,.$$

If appears that the best agreement with data is obtained if $m_\mathrm{X}^{\mathrm{eff}} = 0.63\,\mathrm{GeV}$ for $\mathrm{D}^\pm$-decays and $m_\mathrm{X}^{\mathrm{eff}} = 0.67\,\mathrm{GeV}$ for D^0-, $\overline{\mathrm{D}}^0$-decays.

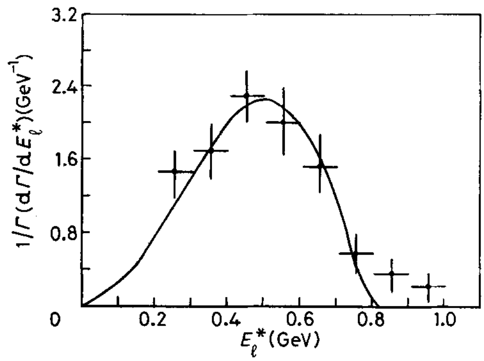

Fig. 4. – Energy spectrum of leptons in $\mathrm{D}\to \ell\nu_\ell \mathrm{X}$ decay. The curve is our result for mixture of D-mesons, experimental points are SLAC data ([45]).

In fig. 4 the comparison of $\Gamma^{-1}\,\mathrm{d}\Gamma/\mathrm{d}E$ calculated in the rest frame of D-meson with experimental data ([45]) is given. The calculation is carried out for a mixture of all charge states of D-meson. The agreement with experiment is rather good in lepton energy region which is essential for our aims. One should note also that the calculation of $\mathrm{D}\to \ell\nu_\ell \mathrm{X}$ decay in terms of the quark model with radiation corrections and taking into account the Fermi motion of heavy quarks in hadron leads to numerically close results for charged-lepton spectra ([45]).

Consider now the semi-leptonic decay of Λ_c^+-barion. In the same manner as above we study at first the process $\Lambda_\mathrm{c}^+ \to \ell\nu_\ell \mathrm{X}$, where X is neutral baryon. As has been shown in ([46]) the matrix element of this decay under the assumption of

([44]) K. YAMADA: *Phys. Rev. D*, **22**, 1676 (1980).
([45]) J. KIRKBY: SLAC-PUB-2231 (1978).
([46]) A. J. BURAS: *Nucl. Phys. B*, **109**, 373 (1976).

$SU(4)$-symmetry and using the hypothesis of vector current conservation can be expressed through the three form factors $f_j(q^2)$:

$$\mathcal{M} = \sqrt{3/2}\,A(c)\,G_F\,\overline{u}(p)[f_1(q^2)\gamma_\mu + if_2(q^2)\sigma_{\mu\nu}q^\nu/m + f_3(q^2)\gamma_\mu\gamma_5]\,u(p_0)j^\mu . \tag{4.9}$$

Here c is Cabibbo factor,

$$A(c=\cos\theta_C) = \sqrt{2/3}\cos\theta_C\,, \qquad A(c=-\sin\theta_C) = -\sin\theta_C\,,$$

p_0 and p are four-momenta of Λ_c^+ and X, $m = m_{\Lambda_c^+} + m_X$. The other notations are the same as above. The concrete forms of q^2-dependence of f_j are given in [46]. Again we use the approximation $f_j = \overline{f}_j = f_j(\overline{q^2})$, $\overline{q^2} = 2(m^2_{\Lambda_c^+} - m^2_X)$. The final expression for lepton energy spectra (ℓ and ν_ℓ) in laboratory frame is written as

$$\begin{cases} W_\zeta^{\Lambda_c^+}(E,E_0) = [1/\Gamma(\Lambda_c^+\to\ell\nu_\ell X)]\,\mathrm{d}\Gamma_\zeta^{\Lambda_c^+}/\mathrm{d}E\,, \\ \mathrm{d}\Gamma_\zeta^{\Lambda_c^+}/\mathrm{d}E = 2(8\pi)^{-3}A^2(c)\,G_F^2\,m^6_{\Lambda_c^+}E_0^{-2}\sum\limits_{i\leq j}\overline{f}_i\,\overline{f}_j\,\varkappa_{ij}^\zeta(x)\,. \end{cases} \tag{4.10}$$

The functions $\varkappa_{ij}^\zeta(x)$ (where $x = E/E_0$, E and E_0 are energies of ℓ and Λ_c) are given in the appendix. They depend on $r = m_X/m_{\Lambda_c^+}$; $i,j = 1,2,3$. The index ζ in (4.10) specifies the type of lepton (muon or neutrino).

The expression for partial decay width in the centre-of-mass frame is

$$\Gamma(\Lambda_c^+\to\ell\nu_\ell X) = 2(8\pi)^{-3}A^2(c)\,G_F^2\,m^5_{\Lambda_c^+}\sum_{i\leq j}\overline{f}_i\,\overline{f}_j\,K_{ij}(r)\,. \tag{4.11}$$

The functions $K_{ij}(r)$ are also given in the appendix.

Passing on to the inclusive decay $\Lambda_c^+\to\mu\nu$ + all we choose the value m_X^{eff} as above, *i.e.* by accordance of calculation (using (4.11)) and experimental data on $B_{\Lambda_c^+}/\tau_{\Lambda_c^+}$ (see table IV). The value chosen is $m_X^{\text{eff}} = 1.26\,\text{GeV}$. The corresponding values of form factors are equal to

$$\overline{f}_1 = 0.991\,, \qquad \overline{f}_2 = 2.170\,, \qquad \overline{f}_3 = 0.805\,.$$

One should note in conclusion that in our approximation the decay spectra of electron and muon (anti) neutrino do not differ from one another. For definiteness we will talk everywhere below about ν_μ and $\overline{\nu}_\mu$.

5 – Results and conclusions.

In the first part of this section we use only modified (nonasymptotical) distribution functions $f_c^{s\,\text{mod}}(x)$ (eq. (3.4c)) which seems to be more satisfactory from the viewpoint of modern accelerator data. The results of muon flux calculations with use of asymptotical distributions (3.4a) are shown on the last several figures together with experimental data points.

Most of the results presented below have been obtained for the F-model of primary cosmic-ray spectrum (see sect. 2). The comparison of differential prompt lepton intensities on sea-level $((\mu^+ + \mu^-)_{pr}$ and $(\nu_\mu + \bar{\nu}_\mu)_{pr})$ calculated for F- and D-variants of primary spectrum and for two values of zenith angle ($\theta = 0°$ and $90°$) is presented in fig. 5 and fig. 6. At energies which are accessible for modern cosmic-ray muon experiments the difference between F- and D-variants is rather small. But at the energy region above few hundred TeV this difference becomes noticeable reaching $\sim 50\%$ (for muons as well as for neutrinos), almost independently of the zenith angle. The steepening of primary spectrum and some other factors ($e.g.$ the energy dependence of fractional moments of D and Λ_c^+

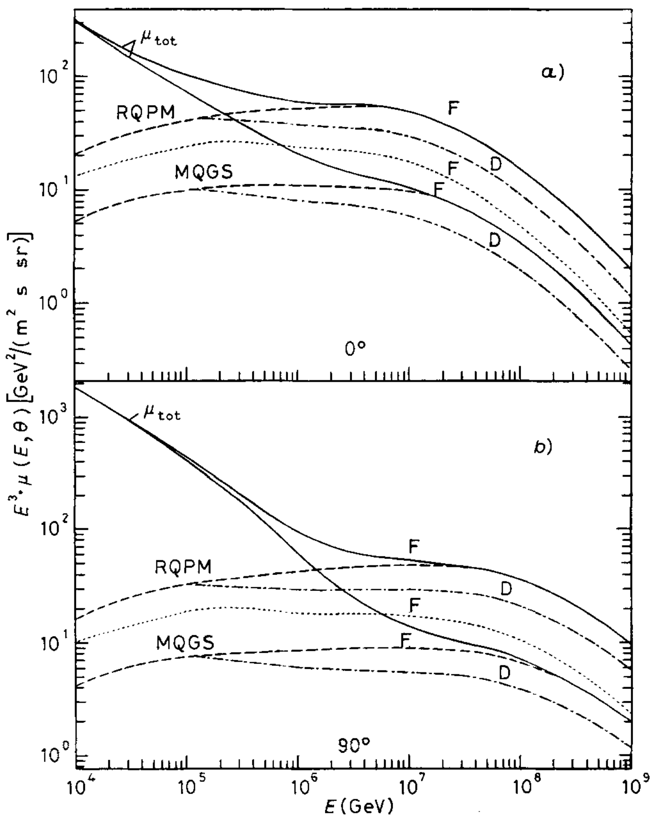

Fig. 5. – Differential energy muon spectra for vertical (a)) and horizontal (b)) directions. Muons from π- and K-decays are considered according to the work ([9]). Prompt muons are calculated in RQPM and MQGS for two variants of primary spectra. Pointed curves: spectra calculated in RQPM under the assumption of Feynman scaling (with the using of $f_c^{s(mod)}(x)$).

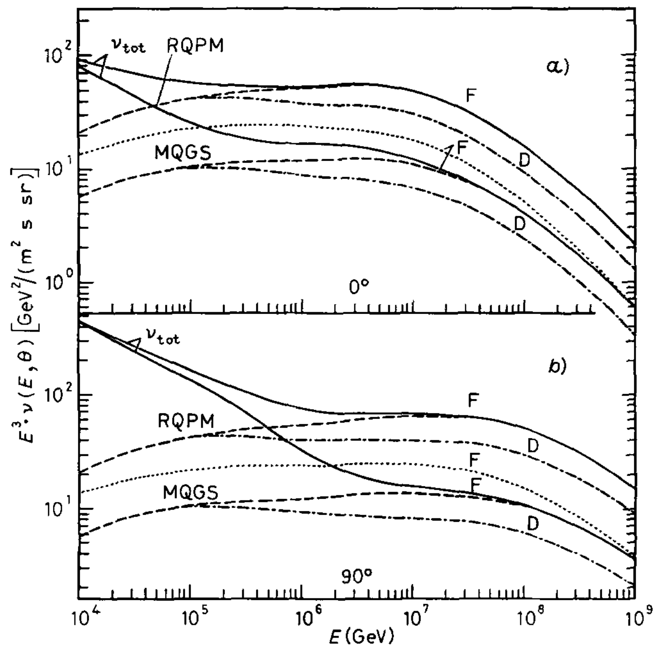

Fig. 6. – Total differential spectra of muon neutrinos and antineutrinos for vertical (*a*)) and horizontal (*b*)) directions. Neutrinos from π- and K-decays are considered according to the work [9]. Prompt neutrinos are calculated in RQPM and MQGS for two variants of primary spectrum. Dotted curves: spectra calculated in RQPM under the assumption of Feynman scaling.

TABLE VII. – *Differential energy spectra of muons for vertical and horizontal directions (in units of* $m^{-2} \cdot s^{-1} \cdot sr^{-1} \cdot GeV^{-1}$*) calculated in two models of charm production.*

E	RQPM		MQGS	
(TeV)	0°	90°	0°	90°
10^1	$2.08 \cdot 10^{-11}$	$1.63 \cdot 10^{-11}$	$5.34 \cdot 10^{-12}$	$1.18 \cdot 10^{-12}$
$3 \cdot 10^1$	$1.16 \cdot 10^{-12}$	$9.18 \cdot 10^{-13}$	$2.94 \cdot 10^{-13}$	$2.32 \cdot 10^{-13}$
10^2	$4.08 \cdot 10^{-14}$	$3.21 \cdot 10^{-14}$	$9.80 \cdot 10^{-15}$	$7.53 \cdot 10^{-15}$
$3 \cdot 10^2$	$1.76 \cdot 10^{-15}$	$1.35 \cdot 10^{-15}$	$3.91 \cdot 10^{-16}$	$2.95 \cdot 10^{-16}$
10^3	$4.98 \cdot 10^{-17}$	$3.99 \cdot 10^{-17}$	$1.03 \cdot 10^{-17}$	$8.14 \cdot 10^{-18}$
$3 \cdot 10^3$	$2.00 \cdot 10^{-18}$	$1.70 \cdot 10^{-18}$	$4.00 \cdot 10^{-19}$	$3.32 \cdot 10^{-19}$
10^4	$4.72 \cdot 10^{-20}$	$4.78 \cdot 10^{-20}$	$9.40 \cdot 10^{-21}$	$9.03 \cdot 10^{-21}$
$3 \cdot 10^4$	$1.18 \cdot 10^{-21}$	$1.66 \cdot 10^{-21}$	$2.42 \cdot 10^{-22}$	$3.07 \cdot 10^{-22}$
10^5	$1.51 \cdot 10^{-23}$	$3.49 \cdot 10^{-23}$	$3.25 \cdot 10^{-24}$	$6.50 \cdot 10^{-24}$
$3 \cdot 10^5$	$2.27 \cdot 10^{-25}$	$8.03 \cdot 10^{-25}$	$5.04 \cdot 10^{-26}$	$1.56 \cdot 10^{-25}$
10^6	$2.01 \cdot 10^{-27}$	$9.62 \cdot 10^{-27}$	$4.44 \cdot 10^{-28}$	$1.96 \cdot 10^{-27}$

TABLE VIII. – *Differential energy spectra of prompt muon neutrino for vertical and horizontal directions (in units of* $m^{-2} \cdot s^{-1} \cdot sr^{-1} \cdot GeV^{-1}$*) calculated in two models of charm production.*

E	RQPM		MQGS	
(TeV)	0°	90°	0°	90°
10^1	$2.10 \cdot 10^{-11}$	$2.10 \cdot 10^{-11}$	$5.72 \cdot 10^{-12}$	$5.72 \cdot 10^{-12}$
$3 \cdot 10^1$	$1.17 \cdot 10^{-12}$	$1.17 \cdot 10^{-12}$	$3.16 \cdot 10^{-13}$	$3.16 \cdot 10^{-13}$
10^2	$4.17 \cdot 10^{-14}$	$4.17 \cdot 10^{-14}$	$1.06 \cdot 10^{-14}$	$1.06 \cdot 10^{-14}$
$3 \cdot 10^2$	$1.79 \cdot 10^{-15}$	$1.81 \cdot 10^{-15}$	$4.31 \cdot 10^{-16}$	$4.35 \cdot 10^{-16}$
10^3	$5.07 \cdot 10^{-17}$	$5.24 \cdot 10^{-17}$	$1.15 \cdot 10^{-17}$	$1.18 \cdot 10^{-17}$
$3 \cdot 10^3$	$2.04 \cdot 10^{-18}$	$2.25 \cdot 10^{-18}$	$4.53 \cdot 10^{-19}$	$4.88 \cdot 10^{-19}$
10^4	$4.85 \cdot 10^{-20}$	$6.43 \cdot 10^{-20}$	$1.08 \cdot 10^{-20}$	$1.35 \cdot 10^{-20}$
$3 \cdot 10^4$	$1.22 \cdot 10^{-21}$	$2.27 \cdot 10^{-21}$	$2.86 \cdot 10^{-22}$	$4.67 \cdot 10^{-22}$
10^5	$1.58 \cdot 10^{-23}$	$4.94 \cdot 10^{-23}$	$4.01 \cdot 10^{-24}$	$1.03 \cdot 10^{-23}$
$3 \cdot 10^5$	$2.40 \cdot 10^{-25}$	$1.18 \cdot 10^{-24}$	$6.42 \cdot 10^{-26}$	$2.61 \cdot 10^{-25}$
10^6	$2.14 \cdot 10^{-27}$	$1.48 \cdot 10^{-26}$	$5.77 \cdot 10^{-28}$	$3.55 \cdot 10^{-27}$

production) leads to essentially nonpower behaviour of μ_{pr} and ν_{pr} spectra on almost all studied energy intervals.

Now we compare the consequences of two models of charm production discussed above (everywhere below the F-variant of primary spectrum will be used). The differential energy spectra of $(\mu^+ + \mu^-)_{pr}$ and $(\nu_\mu + \bar{\nu}_\mu)_{pr}$ at sea-level calculated in RQPM and MQGS for $\theta = 0°$ and $\theta = 90°$ are shown in tables VII, VIII and fig. 5, 6. On the same figures the prompt lepton spectra calculated with the assumption of constant (*i.e.* independent of energy) cross-section $d\sigma_{fi}^A/dx$ and the total spectra ($\mu_{tot} = \mu_{\pi,K} + \mu_{pr}$ and $\nu_{tot} = \nu_{\pi,K} + \nu_{pr}$) are also shown. Here (and below) the spectra of $\mu_{\pi,K}$ and $\nu_{\pi,K}$ taken from (9) are used. The parameters of

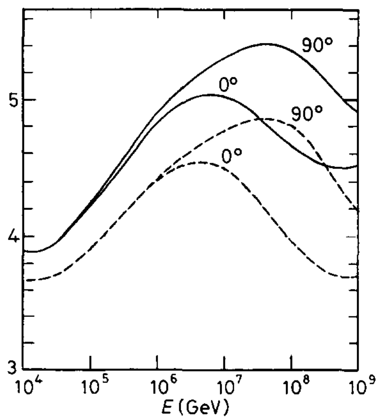

Fig. 7. – Comparison of energy dependences of prompt muon fluxes (solid curves) and neutrinos fluxes (dashed curves) calculated in RQPM and MQGS.

nuclear cascade used in [9] are quite similar to ours (in particular, their primary spectrum is close to our F-variant).

In fig. 7 the comparison of energy dependences of prompt lepton fluxes calculated in RQPM and MQGS is presented. As one can see the predictions of these models differ by a factor $3 \div 5$ for μ_{pr} and ν_{pr} (the exact value depends on energy and zenith angle). The largest differences are for $E_{\mu,\nu} \sim (2 \div 5) \cdot 10^3$ TeV, $\theta = 0°$ and for $E_{\mu,\nu} \sim (3 \div 5) \cdot 10^4$ TeV, $\theta = 90°$, *i.e.* in the energy region which is accessible for DUMAND detectors. The main cause of this difference is in the comparatively steep behaviour of D-meson production cross-sections $d\sigma/dx$ (as functions of x) in MQGSs case (see sect. 3). Besides that we assume in the MQGSs calculations (in contrast with RQPMs case) that the A-dependence of cross-sections is characterized by constant α. For all charmed particles the same value ($\alpha = 0.72$) was taken.

The relative contribution of Λ_c^+-particles in μ- and ν-fluxes is essentially different in two models: in RQPM this contribution is very small even for very high energies while in MQGS it is comparable with the one of D-meson for $E \lesssim 10^3$ TeV and becomes dominant for $E \gtrsim 10^4$ TeV. One should note that even the use of old experimental inclusive spectrum of Λ_c^+ (which is more flat than the RQPM prediction) does not change the relative yield of μ_{pr} very much (see ref. [12]).

The role of reactions $\pi A \rightarrow D(\Lambda_c^+)X$ weakly depends on zenith angle and quickly decreases with energy in both models. At the same time at the energies

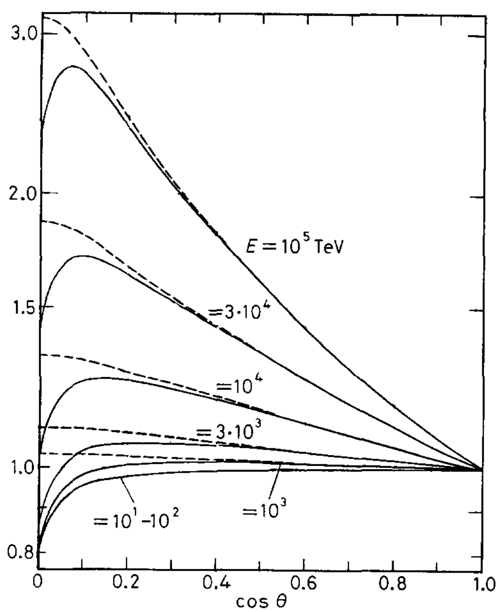

Fig. 8. – Zenith-angle distributions of prompt $(\mu^+ + \mu^-)$-fluxes (solid curves) and $(\nu_\mu + \bar{\nu}_\mu)$-fluxes (dashed curves) calculated in RQPM. Numbers at curves are the energies in TeV.

$E_\mu \lesssim 200\,\text{TeV}$ the contribution of these processes is not negligible ($\gtrsim 12\%$) and should be taken into account.

The zenith-angle distributions of prompt muons and neutrinos (fig. 8) are qualitatively similar (with a proper change of energy scale) to the distributions of usual (π, K)-leptons. At the «low» energy region, $E \lesssim 10^4\,\text{TeV}$ the spectra of μ_{pr} and ν_{pr} are almost isotropical independently of model. For higher energies the zenith-angle distributions in RQPM and MQGS are similar (the differences are smaller than $\sim 16\%$ for μ_{pr} and $\sim 22\%$ for ν_{pr}). The analysis shows the weak dependence of the distributions on the value of parameter H^* even for large zenith angles.

In fig. 9 and 10 the comparison of our prompt lepton differential spectra with the corresponding results of other authors is given. The vertical and horizontal spectra of nonprompt leptons shown in these figures are taken from (1,9). As one can see from fig. 9 the yields into total muon flux from the usual and prompt muons become equal at energies ~ 170 TeV (RQPM) and ~ 1000 TeV (MQGS) for vertical directions and correspondingly ~ 1300 TeV and ~ 6000 TeV for horizontal direction. The neutrino fluxes of usual and prompt neutrino (fig. 10) become equal at energies ~ 32 TeV (RQPM) and ~ 170 TeV (MQGS) for vertical direction, and, correspondingly, ~ 340 TeV and ~ 1700 TeV for horizontal.

The predictions of RQPM for muon energies $(1 \div 100)$ TeV are close to the results of (6) (the curve IKK1 in fig. 9, corresponding to model 1 of (6)). The predictions of MQGS for muons are more similar to the results of «central» model

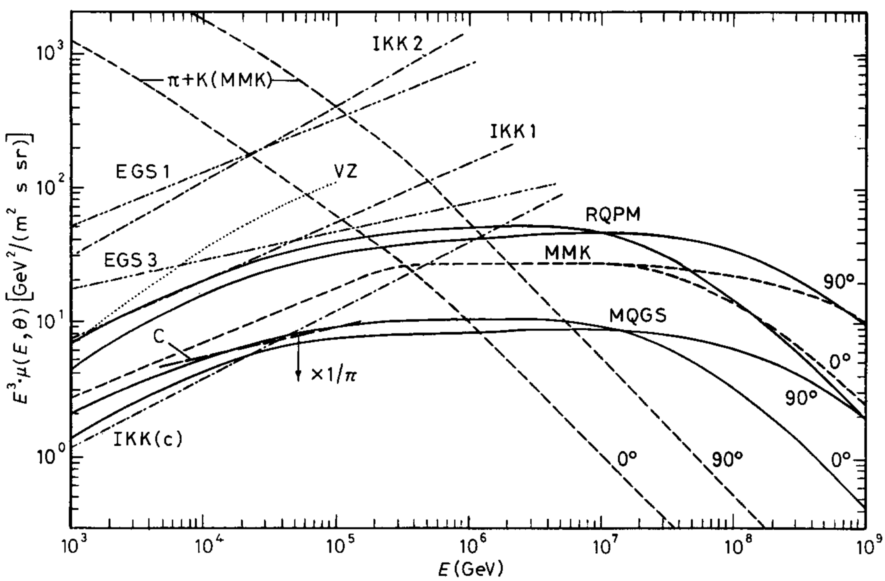

Fig. 9. – Comparison of differential energy spectra of prompt muons, calculated by various authors. Solid curves: calculation of this work in RQPM and MQGS, curve V (1), VZ (2), EGS1 and EGS3: models 1 and 3 of work (3), IK (4), BDZ (5), IKK(c), IKK1 and IKK2 models of work (6), C (7), MMK (9).

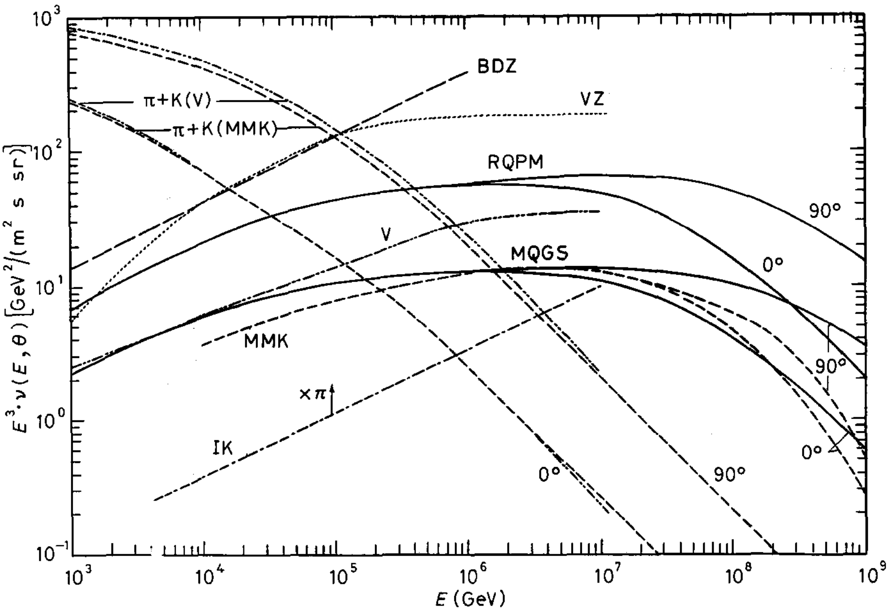

Fig. 10. – Comparison of differential energy spectra of prompt neutrinos calculated by various authors. Solid curves: calculation of this work in RQPM and MQGS; curve V (1), VZ (2), IK (4), BDZ (5), MMK (9).

of the same authors (6) (IKK(c) in fig. 9). One must note that the steepening of primary spectra and the muon energy losses in the atmosphere have not been taken into account in (6). At very high energies our RQPM result is close to the result of (9) (see also (8)). The prompt neutrino spectra calculated in RQPM are essentially different from the results of other works, and the corresponding MQGS results are similar to the spectra (1) (for $E \lesssim 10^2$ TeV) and (9) (for $E \gtrsim 10^2$ TeV). The zenith-angle dependence of prompt neutrino flux at superhigh energies (in both our models) is qualitatively different from the same dependence obtained in (9). The main source of this contradiction is the difference in semileptonic decay spectra of D and Λ_c^+ used by us (see sect. 4) and authors of (9). One must note also that our previous work (12) contains some distinctions from the present one (the different way of the break of scaling, the slightly different values of some parameters) but the final results are quite close.

In the energy region $5\,\text{TeV} \lesssim E \lesssim 5 \cdot 10^3\,\text{TeV}$ the differential prompt muon $(\mu^+ + \mu^-)$ and neutrino $(\nu_\mu + \bar{\nu}_\mu)$-spectra for vertical direction at sea-level can be approximated (with accuracy better than $(3 \div 4)\%$) by the following formulae:

$$\mu_{\text{pr}}(E, 1030\,\Gamma/\text{cm}^2, 0°) = \mu_0 (E_\text{b}/E)^{\gamma_\mu + 1} [1 + (E_\text{b}/E)^{\gamma_\mu}]^{-a_\mu}, \tag{5.1}$$

$$\nu_{\text{pr}}(E, 1030\,\Gamma/\text{cm}^2, 0°) = \nu_0 (E_\text{b}/E)^{\gamma_\nu + 1} [1 + (E_\text{b}/E)^{\gamma_\nu}]^{-a_\nu}. \tag{5.2}$$

Here $E_b = 10^5\,\text{GeV}$ and the other parameters are equal to

$$\text{in RQPM}\quad \begin{cases} \mu_0 = 4.53 \cdot 10^{-14} (\text{m}^2 \cdot \text{s} \cdot \text{sr} \cdot \text{GeV})^{-1}, & \gamma_\mu = 1.96, \quad a_\mu = 0.152 \\ \nu_0 = 4.65 \cdot 10^{-14} (\text{m}^2 \cdot \text{s} \cdot \text{sr} \cdot \text{GeV})^{-1}, & \gamma_\nu = 1.96, \quad a_\nu = 0.157 \end{cases}$$

$$\text{in MQGS}\quad \begin{cases} \mu_0 = 1.09 \cdot 10^{-14} (\text{m}^2 \cdot \text{s} \cdot \text{sr} \cdot \text{GeV})^{-1}, & \gamma_\mu = 2.02, \quad a_\mu = 0.165 \\ \nu_0 = 1.19 \cdot 10^{-14} (\text{m}^2 \cdot \text{s} \cdot \text{sr} \cdot \text{GeV})^{-1}, & \gamma_\nu = 2.01, \quad a_\nu = 0.165. \end{cases}$$

The approximation (5.1) is valid with the same accuracy also for zenith angles $\theta \leqslant 80°$ in the energy interval $(10 \div 10^3)\,\text{TeV}$ (*i.e.* in the region of isotropy). For $\theta > 80°$ formula (5.1) gives the upper limit of μ_{pr} spectra. The dependence (5.2) is valid for all zenith angles in the energy interval $(5 \div 10^3)\,\text{TeV}$.

From (5.1) and (5.2) we obtain the following expressions for integral spectra

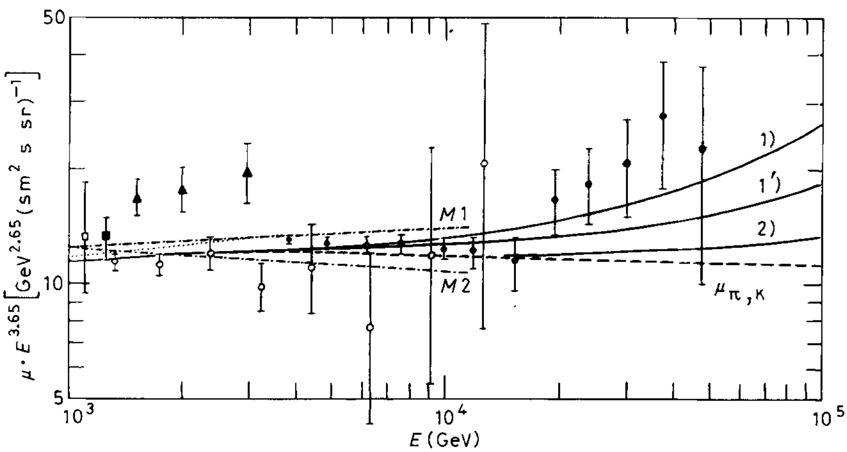

Fig. 11. – Differential energy muon spectrum on sea-level for vertical. Experimental data: □ Durgapure ([47]), ■ and ······ Nottingham ([48]), ▲ Durham ([49]), curves –·–·– results of two experiments under Mont Blanc (M1 - French-U.S., M2 - Italian) ([50]), points ○ (Artemovsk) and ● (MSU) are taken from ([11]). 1) our calculation in RQPM with asymptotical distribution of c-quarks, 1′) RQPM with modified distribution of c-quarks, 2) MQGS. Dashed curve: (π, K)-muon spectrum according to ([9]); $\theta = 0°$.

([47]) B. C. NANDI and M. S. SINDA: *J. Phys. A*, **5**, 1384 (1972).
([48]) B. C. RASTIN: *J. Phys. G*, **10**, 1609 (1984).
([49]) M. G. TOMPSON, R. THORNLEY, M. R. WHALLEY and A. W. WOLFENDALE: *Proceedings of the XV International Cosmic Rays Conference*, Vol. **6** (Plovdiv, 1977). p. 21.
([50]) L. BERGAMASCO, A. CASTELLINA, B. D'ETTORE PIAZZOLI, G. MANNOCCHI, P. PICCHI, S. VERNETTO and H. BILOKON: *Nuovo Cimento C*, **6**, 569 (1983).

of prompt leptons at sea-level ($5\,\text{TeV} \leqslant E \leqslant 10^3\,\text{TeV}$):

$$\mu_{\text{pr}}(>E) = \frac{\mu_0 E_{\text{b}}}{\gamma_\mu(1-a_\mu)}\left\{\left[1+\left(\frac{E_{\text{b}}}{E}\right)^{\gamma_\mu}\right]^{1-a_\mu} - 1\right\},$$

$$\nu_{\text{pr}}(>E) = \frac{\nu_0 E_{\text{b}}}{\gamma_\nu(1-a_\nu)}\left\{\left[1+\left(\frac{E_{\text{b}}}{E}\right)^{\gamma_\nu}\right]^{1-a_\nu} - 1\right\}.$$

In fig. 11 the comparison of our results for differential spectra at sea-level with experimental data (11,47-50) is presented. The data of magnetic spectrometers (47-49), underground array situated under Mont Blanc (50), scintillation detectors in Artemovsk (51) and X-ray emulsion chambers (11) are used. Spectra of (50) (lines M1 and M2 in fig. 11) are obtained from the analysis of muon absorption curve. The experimental points of (11) for vertical spectrum have been obtained by the calculation from global flux with assumption $R = 0.2\%$ (R is the prompt muon fraction). One should note that the conclusion about the value of R obtained in (11) by analysis of angular distribution is rather uncertain ($R = (0.2 \pm 0.2)\%$) so the systematic discrepancy between our curves and the experimental points in fig. 11 cannot be treated too seriously.

One should note that the recent data of measurements under Mont Blanc (NUSEX) (52) give only the upper limit of prompt muon flux:

$$\mu_{\text{pr}}(E) \leqslant 1.5\,E^{-2.78}\,\text{m}^{-2}\,\text{s}^{-1}\,\text{sr}^{-1}\,\text{GeV}^{-1}.$$

This limit does not contradict our MQGS calculation but essentially disagree with the prediction of RQPM. On the other hand the conclusion of (52) (that the energy at which prompt muon flux reaches the conventional one is about 200 TeV) is in qualitative agreement with RQPM results.

In fig. 12 (*a*)-*c*)) the comparison of calculated integral muon spectrum at sea-level with experiment is presented. The results of HAS experiment (Akeno) were taken from (4) (in (4) the experimental points are normalized on the calculated spectrum). The most interesting results are obtained in underground experiments in KGF and Baksan, in which the flattening of muon spectrum at E_μ more than few tens of TeV is clearly seen. The KGF data are taken from (10)

(51) F. F. Khalchukov, E. V. Korolkova, V. A. Kudryavtsev, A. S. Milgin, O. G. Ryazhskaya and G. T. Zatsepin: *Proceedings of the XIX International Cosmic Rays Conference*, Vol. 8 (La Jolla, 1985), p. 12.

(52) G. Battistoni, C. Bloise, P. Campana, V. Chiarella, A. Ciocio, R. Iarocci, G. P. Murtas, G. Nicoletti, L. Satta, M. Rollier, L. Zanotti, D. C. Cundy and M. Price: *Proceedings of the XX International Cosmic Rays Conference*, Vol. 9 (Moscow, 1987), p. 195.

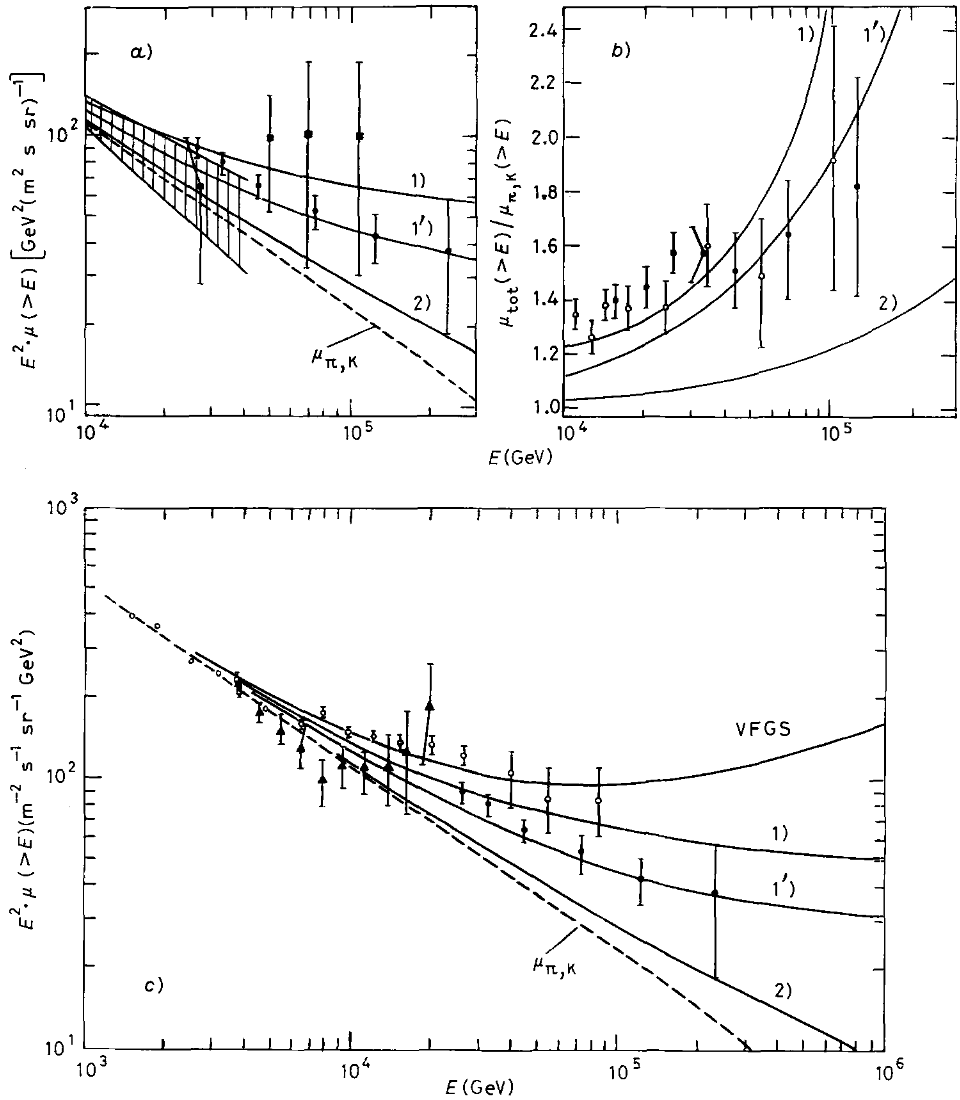

Fig. 12. – Comparison of integral muon spectrum calculated in RQPM and MQGS with experimental data. Curves 1), 1′) and 2) have the same meaning as the ones given in fig. 11. Curve VFGS: calculation of Volkova *et al.* ([53]). Experimental data: *a*) ● KGF ([10]), ■, ||||||||||||||| Akeno ([4]); *b*) ● 7000 hg/cm^2, □ 6045 hg/cm^2 KGF ([54]) (in the last work data are normalized on the theoretical spectrum of π, K-muons; *c*) ● KGF ([10]), ○ Baksan ([55]), ▲ Baksan ([56]).

([53]) L. V. Volkova, W. Fulgione, P. Galeotti and O. Saavedra: *Nuovo Cimento C*, **10**, 465 (1987).

([54]) V. S. Narasimham: *Proceedings of the XX International Cosmic Rays Conference*, Vol. 8 (Moscow, 1987), p. 288 (Rapporteur Talks).

([55]) Yu. M. Andreyev, V. I. Gurentsov and I. M. Kogai: *Proceedings of the XX International Cosmic Rays Conference*, Vol. **6** (Moscow, 1987), p. 200.

([56]) V. N. Bakatanov, Yu. P. Novosel'tsev, R. V. Novosel'tseva, A. M. Semenov, Yu. V. Sten'kin and A. E. Chudakov: to be published in *Izv. Akad. Nauk SSSR, Ser. Fiz.*

and ([54]), and Baksan data from ([55,57]) (in these experiments for prompt muon flux calculated the muon absorption curve and angular distributions have been used, with assumption $R = 0.15\%$) and ([56]) (where the muon flux is calculated from the measured electromagnetic cascade spectrum). As one can see from the figure, the results of our calculations in RQPM for two variants of c-quark sea distributions lie between experimental points of Baksan and KGF. One can conclude that the recombination quark-parton model of charmed hadron production is preferential.

* * *

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Appendix

Let us write expressions for functions $\varkappa_{ij}^{\zeta}(x)$ and $K_{ij}(r)$ entering eq. (4.10) and eq. (4.11), correspondingly. The value of parameter ζ is equal to $+1$ for the spectra of muons (electrons) and -1 for the spectra of neutrino (antineutrino). The dependence on ζ enters the only function: $\varkappa_{13}^{\zeta}(x)$. After rather complicated but standard calculations we obtain:

$$\begin{aligned}\varkappa_{11}(x) = {} & 11/6 - 3r - 21r^2/2 - 12r^3 + 9r^4/2 + 9r^5 + 7r^6/6 + 6r^3(2+r)\,x - \\ & - 3(3 - 2r - 5r^2)\,x^2/2 + 8x^3/3 + r^4(3 + 6r + r^2)/(1-x) - r^6/(1-x)^2 + \\ & + r^3(12 + 9r + 6r^2 - r^3)\ln((1-x)/r^2)\,,\end{aligned}$$

$$\begin{aligned}\varkappa_{12}(x) = {} & 5/3 - 6r - 9r^2 - 24r^3 + 3r^4 + 18r^5 - 5r^6/3 + 2r^4(3 + 6r + r^2)/(1-x) - \\ & - 2r^6/(1-x)^2 + 2r^3(12 + 3r + 6r^2 - r^3)\ln((1-x)/r^2)\,,\end{aligned}$$

$$\begin{aligned}\zeta\varkappa_{13}^{\zeta}(x) = {} & -1/3 + 3r^2 - 3r^4 - 17r^6/3 - 12r^4 x + 3(1 - 3r^2)\,x^2 - 8x^3/3 + \\ & + 2r^4(3 + r^2)/(1-x) - 2r^6/(1-x)^2 - 2r^4(3 + r^2)\ln((1-x)/r^2)\,,\end{aligned}$$

$$\begin{aligned}\varkappa_{22}(x) = {} & (1+r)^{-2}(2/3 - 4r/3 - 23r^2/3 - 24r^3 - 21r^4 - 40r^6/3 + \\ & + 22r^7/3 - 4r^8/3 + 2r^3(6 + 9r + 6r^2 - r^3)\,x - 3(1/2 - 2r^2 - 2r^3 + r^6/2)\,x^2 + \\ & + 4(1 + r - r^2)\,x^3/3 - x^4/2 + r^4(3 + 12r - 14r^2 + 8r^3 + r^4)/(1-x) - \\ & - r^6(1+r)^2/(1-x)^2 + r^3(12 + 21r - 24r^2 + 10r^3 + 4r^4 - r^5)\ln((1-x)/r^2))\,,\end{aligned}$$

([57]) Yu. M. Andreev, V. I. Gurentsov, I. M. Kogai and A. E. Chudakov: to be published in *Izv. Akad. Nauk SSSR, Ser. Fiz.*

$$\varkappa_{23}^{\zeta}(23) = \varkappa_{13}^{\zeta}(x),$$

$$\varkappa_{33}(x) = 11/6 + 3r - 21r^2/2 + 12r^3 + 9r^4/2 - 9r^3 + 7r^6/6 - 6r^3(2-r)x - \\ - 3(3/2 + r - 5r^2/2)x^2 + 8x^3/3 + r^4(3 - 6r + r^2)/(1-x) - \\ - r^6/(1-x)^2 - r^3(12 - 9r + 6r^2 + r^3)\ln((1-x)/r^2),$$

where $r = m_X/m_{\Lambda_c^+}$, $x = E/E_0$.

Nonzero functions $K_{ij}(r)$ are equal to

$$K_{11} = 1 - 2r - 8r^2 - 18r^3 - 18r^5 - 8r^6 + 2r^7 - r^8 - 24r^3(1 + r^2 + r^3)\ln r,$$

$$K_{12} = 1 - 4r - 8r^2 - 36r^3 - 36r^5 - 8r^6 + 4r^7 - r^8 - 24(2 + r + 2r^2)\ln r,$$

$$K_{22} = (1+r)^{-2}(2/5 - r - 6r^2 - 28r^3 - 32r^6 - 28r^7 + 6r^8 + \\ + r^9 - 2r^{10}/5 - 24r^3(1 + 2r + 3r^2 + 2r^3 - r^4)\ln r),$$

$$K_{33} = 1 + 2r - 8r^2 + 18r^3 - 18r^5 + 8r^6 - 2r^7 - r^8 - 24r^3(1 - r + r^2)\ln r.$$

● RIASSUNTO (*)

Si calcolano gli spettri d'energia e le distribuzioni dell'angolo di zenith dei muoni dei raggi cosmici, dei neutrini e degli antineutrini della generazione istantanea per un intervallo d'energia $(1 \div 10^6)$ TeV. Per i calcoli delle sezioni d'urto differenziali della produzione di $D^\pm$, D^0, $\overline{D}^0$ e Λ_c nelle interazioni $\mathcal{N}\mathcal{N}$ e $\pi\mathcal{N}$ si usa il modello di ricombinazione quark-partoni (RQPM). Si tiene conto degli effetti nucleari usando il modello additivo dei quark e il modello ottico del nucleo. Si effettua un confronto dettagliato dei risultati ottenuti nel RQPM con le previsioni corrispondenti del modello della stringa di quark-gluoni. Si considera la dinamica dei decadimenti semi leptonici di Λ_c e D e le perdite d'energia dei muoni nell'atmosfera. Si effetuano i calcoli delle cascate adroniche nell'atmosfera tenendo conto della crescita con l'energia delle sezioni d'urto totali anelastiche adrone-nucleo, la rapida crescita degli spettri dei raggi cosmici primari e la rigenerazione dei pioni. Si confrontano i nostri calcoli con i dati sperimentali e con i calcoli di altri autori.

(*) *Traduzione a cura della Redazione.*

Прямые лептоны в космических лучах

Резюме. — Рассчитаны энергетические спектры и зенитно-угловые распределения космических мюонов, нейтрино и антинейтрино прямой генерации при энергиях $(1 \div 10^6)$ ТэВ. для расчета дифференциальных сечений рождения $D^\pm$, D^0, $\bar{D}^0$ и Λ_c^+ в $\mathcal{N}\mathcal{N}$- и $\pi\mathcal{N}$-взаимодействиях использована рекомбинационная кварк-партонная модель (РКПМ). Пересчет на ядро сделан в рамках аддитивной кварковой модели и оптической модели ядра. Проведено подробное сопоставление реультатов, полученных в рамках РКПМ, с предсказаниями модели кварк-глюонных струн (МКТС). Учтена динамика полулептонных распадов D и Λ_c^+ и энергетические потери мюонов в воздухе. При расчете $\pi\mathcal{N}$-каскада в атмосфере приняты во внимание рост с энергией полных неупругих сечений взаимодействия адронов с ядрами, излом спектра первичного космического излучения, процессы регенерации пионов. Дано сравнение расчетов с экспериментальными данными и с расчетами других авторов.